

Just-in-time purchasing: an integrated inventory model involving deterministic variable lead time and quality improvement investment

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Nowadays supply chain management is a popular practice in manufacturing systems, and just-in-time (JIT) production plays a crucial role in supply chain environments. Companies are using JIT production to gain and maintain a competitive advantage. The characteristics of JIT systems are consistent high quality, small lot sizes, frequent delivery, short lead time, and close supplier ties. This paper presents an integrated inventory model to minimize the sum of the ordering/setup cost, holding cost, quality improvement investment and crashing cost by simultaneously optimizing the order quantity, lead time, process quality and number of deliveries while the probability distribution of the lead time demand is normal. This integrated inventory model is useful particularly for JIT inventory systems where the vendor and the purchaser form a strategic alliance for profit sharing.

1. Introduction

The JIT system plays an important role in present supply chain management. One of the major tasks of maintaining the competitive advantages of JIT production is to compress the lead time needed to perform activities associated with delivering high-quality products to customers. In the dynamic, competitive environment, successful companies have devoted considerable attention to reducing inventory cost and lead time and improving quality simultaneously.

In recent years, companies have found that there are substantial benefits from establishing a long-term sole-supplier relationship with a supplier (Martinich 1997). In the JIT environment, a close cooperation exists between supplier and purchaser to solve problems together, and thus to maintain stable, long-term relationships. Since both purchaser and supplier may benefit from the negotiation process, the two sides must then negotiate to determine how to divide the savings (Thomas and Griffin 1996). The advantages of the integrated inventory model include improved quality, lowered inventory cost, technology sharing and reduction of lead time.

In the production environment, lead time is an important element in any inventory management system. Traditionally, most of the literature dealing with inventory problems treated lead time as a prescribed constant (Kim and Park 1985, Ravichandran 1995) or a stochastic variable (Foote *et al.* 1988), which therefore is not subject to control. However, in many practical situations, lead

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time can be reduced by an additional crashing cost, and hence is controllable. By shortening the lead time, a company can lower the safety stock, reduce the loss caused by stockout, improve customer service level and thus increase the competitive edge in business (Ouyang and Wu 1997). The reduction of lead time mainly consists of the following components: order preparation, order transit, supplier lead time and delivery time (Tersine 1982).

Liao and Shyu (1991) initiated a study on lead time reduction by presenting a continuous review model in which the order quantity was predetermined and lead time was the only decision variable. Ben-Daya and Raouf (1994) extended the model of Liao and Shyu (1991) by allowing both lead time and the order quantity to be decision variables. They derived the optimal lead time and optimal order quantity that minimized the sum of the ordering cost, holding cost and lead-time crashing cost. Ouyang *et al.* (1996) generalized the Ben-Daya and Raouf (1994) model by allowing shortages with a mixture of backorders and lost sales.

The integrated model between supplier and purchaser for improving the performance of inventory control has attracted a great deal of attention from researchers. Goyal (1977) suggested a joint economic lot-size model where the objective was to minimize the total relevant costs for both the vendor and the buyer. Banerjee (1986) presented a joint economic-lot-size model where a vendor produced to order for a purchaser on a lot-for-lot basis under deterministic conditions. Goyal (1988) generalized the Banerjee model (1986) by relaxing the assumption of the lot-for-lot policy of the vendor. Lu (1995) presented a model for one-vendor one-buyer problems, and developed a heuristic approach for the one-vendor multi-buyer case. The model is an improvement over the models of Banerjee (1986) and Goyal (1988). Ha and Kim (1997) addressed an integrated lot-splitting model of facilitating multiple shipments in small lots and compared it with the existing approaches in a simple JIT environment. Gurnani (2001) presented quantity discount pricing models with different ordering structures in a system consisting of a single supplier and heterogeneous buyers. Woo *et al.* (2001) addressed an integrated inventory model where a single vendor purchases and processes raw materials in order to deliver finished items to multiple buyers. Khan and Sarker (2002) proposed a two-stage integrated inventory system to incorporate the JIT concept in the conventional joint batch-sizing problem. Kelle *et al.* (2003) studied quantitative models of establishing and negotiating buyer-supplier partnerships, and presented the two typical cases of supplier dominance with large production lot sizes and shipment sizes, and purchaser dominance with small, frequent shipments. David and Eben-Chaime (2003) analysed aspects of the relationships between a purchaser and a vendor in an attempt to answer how far they should go in terms of lot sizes and delivery frequency.

The integrated model can contribute significantly to improve the vendor-purchaser relationship. The success and resulting performance of the integrated model is based upon the cooperation between the purchaser and supplier; for example, the integrated model practices characterized by a sole-supplier base whose firms are located close to the buyer's plant, make frequent deliveries, and are considered long-term partners with the buying company (Gunasekaran 1999). When establishing a long-term relationship, it is important that the purchaser selects the vendors that have consistently exhibited high levels of quality and delivery reliability (Schonberger and Ansari 1984). Several researchers have shown that in integrated models, one partner's gain exceeds the other partner's loss. Thus, the net benefit can be shared by both parties in some equitable fashion (Goyal and Gupta 1989).

Pan and Yang (2002) presented an integrated supplier–purchaser model focused on the benefit from lead-time reduction while excluding quality-related issues. However, defective items are often and inevitably produced in real production systems. These defective items must be rejected, repaired, reworked, or, if they have reached the customer, refunded. In all cases, substantial costs are incurred (Ouyang *et al.* 2002). Porteus (1986) and Rosenblatt and Lee (1986) first presented the significant relationship between quality imperfection and lot size. Following Porteus (1986), there is abundant quality improvement related literature; for example, see Keller and Noori (1988), Hong and Hayya (1995), Ouyang and Chang (2000) and Ouyang *et al.* (2002).

Since it is more appropriate to take the quality-related cost into account in determining the optimal ordering policy, this paper presents an integrated inventory model for minimizing the sum of the ordering/setup cost, holding cost, quality improvement and crashing costs by simultaneously optimizing the order quantity, the lead time, the process quality and the number of deliveries when the probability distribution of the lead time demand is normal. The advantage of such an integrated model over the traditional model is illustrated by a numerical example.

2. Notation and assumptions

To establish the proposed model, the following notation is used:

- D = average demand per year;
- P = production rate;
- Q = contract quantity;
- A = purchaser's ordering cost per order;
- S = vendor's setup cost per setup;
- L = length of lead time;
- C_V = unit production cost paid by the vendor;
- C_P = unit purchase cost paid by the purchaser;
- r = annual inventory holding cost per dollar invested in stocks;
- i = the fractional per unit time opportunity cost of capital.

The assumptions made in the paper are as follow:

1. The product is manufactured with a finite production rate P , and $P > D$.
2. The demand X during the lead time L follows a normal distribution with mean DL and standard deviation σ .
3. The reorder point (ROP) equals the sum of the expected demand during lead time and the safety stock, that is, the reorder point equals $ROP = \mu L + k\sigma\sqrt{L}$, where k is known as the safety factor.
4. Inventory is continuously reviewed and replenished.
5. The out-of-control probability θ is a continuous decision variable, and is described by a logarithmic investment function. The quality improvement and capital investment is represented by $q(\theta) = q\ln(\theta_0/\theta)$ for $0 < \theta \leq \theta_0$, where θ_0 is the current probability that the production process can go out of control, and $q = 1/\xi$, with ξ denoting the percentage decrease in θ per dollar increase in $q(\theta)$. The application of the logarithmic function on quality improvement and capital investment has been proposed by many authors, for example, Porteus (1986), Hong and Hayya (1995), Ouyang and Chang (2000) and Ouyang *et al.* (2002).

6. The lead time L has n mutually independent components and these components are crashed one at a time starting with the one of least crashing cost per unit time, and so on.

3. A basic model

The expected annual total cost of an integrated inventory model with normally distributed lead-time demand for minimizing the sum of the ordering cost, holding cost and crashing cost can be expressed as (Pan and Yang 2002):

$$\begin{aligned}
 JTEC(Q, L, m) &= \frac{D}{Q} \left[A + \frac{S}{m} + R(L) \right] + \frac{Q}{2} r \left[\left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) C_V + C_P \right] + r C_P k \sigma \sqrt{L} \\
 &= \frac{D}{Q} \left[A + \frac{S}{m} + \left\{ c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j) \right\} \right] \\
 &\quad + \frac{Q}{2} r \left[\left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) C_V + C_P \right] + r C_P k \sigma \sqrt{L} \quad (1)
 \end{aligned}$$

where m is an integer representing the number of shipments of the item delivered to the purchaser, and a_i , b_i , c_i are the minimum duration, normal duration and crashing cost per unit time, respectively, of the i th component of lead time. Let $\sum_{i=1}^n a_i \leq L \leq \sum_{i=1}^n b_i$, and let L_i be the length of the i th component of the lead time crashed to its minimum duration. Then L_i can be expressed as $L_i = \sum_{i=1}^n b_i - \sum_{j=1}^i (b_j - a_j)$, $i = 1, 2, \dots, n$. Also let $R(L)$ denote the lead-time crashing cost per cycle for a given $L \in [L_i, L_{i-1}]$, and $R(L) = c_i[L_{i-1} - L] + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

In order to include the essence of an imperfect production process, consider the assumption made in the model proposed by Porteus (1986). The integrated inventory model is designed for vendor production situations in which, once an order is placed, production begins and a constant number of units is added to the inventory each day until the production run has been completed. The vendor produces the item in the quantity mQ with a given probability of θ that the process can go out of control. Porteus (1986) suggested the expected number of defective items in a run of size mQ can be evaluated as $m^2 Q^2 \theta / 2$. Suppose g is the cost of replacing a defective unit, and the production quantity for the supplier in a lot of mQ . Then its expected defective cost per year is given by $gmQD\theta/2$.

Hence, the total expected annual cost incorporating the defective cost per year can be represented by

$$TC(Q, m, L) = JTEC(Q, m, L) + \frac{gmQD\theta}{2} \quad (2)$$

4. Investment in quality improvement

Based on equation (2), we wish to study the effect of investment on quality improvement. Consequently, the objective of the integrated model is to minimize the sum of the ordering/setup cost, holding cost, quality improvement and crashing cost by simultaneously determining the optimal values of Q , m , θ and L , subject to

the constraint that $0 < \theta \leq \theta_0$. Thus, the total relevant cost per year is

$$\begin{aligned}
 TRC(Q, m, \theta, L) &= TC(Q, m, L) + iq \ln \frac{\theta_0}{\theta} \\
 &= \frac{D}{Q} \left[A + \frac{S}{m} + R(L) \right] + \frac{Q}{2} \left[\left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) rC_V + rC_P + gmD\theta \right] \\
 &\quad + rC_P k \sigma \sqrt{L} + iq \ln \frac{\theta_0}{\theta} \tag{3}
 \end{aligned}$$

for $0 < \theta \leq \theta_0$, where i is the fractional opportunity cost of capital per unit time.

Therefore, the problem under study can be formulated as the following non-linear programming model:

$$\begin{aligned}
 &\text{Minimize } TRC(Q, m, \theta, L) \\
 &= \frac{D}{Q} \left[A + \frac{S}{m} + R(L) \right] + \frac{Q}{2} \left[\left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) rC_V + rC_P + gmD\theta \right] \\
 &\quad + rC_P k \sigma \sqrt{L} + iq \ln \frac{\theta_0}{\theta} \tag{4}
 \end{aligned}$$

Subject to $0 < \theta \leq \theta_0$

In order to find the minimum cost for this non-linear programming problem, ignore the constraint $0 < \theta \leq \theta_0$ for the moment and minimize the total relevant cost function over Q , θ and L with classical optimization techniques by taking the first partial derivatives of $TRC(Q, m, \theta, L)$ with respect to Q , θ and L as follows:

$$\begin{aligned}
 &\frac{\partial TRC(Q, m, \theta, L)}{\partial Q} \\
 &= -\frac{D}{Q^2} \left(A + \frac{S}{m} + R(L) \right) + \frac{1}{2} \left[\left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) rC_V + rC_P + gmD\theta \right] \tag{5}
 \end{aligned}$$

$$\frac{\partial TRC(Q, m, \theta, L)}{\partial \theta} = \frac{gmDQ}{2} - \frac{iq}{\theta} \tag{6}$$

$$\frac{\partial TRC(Q, m, \theta, L)}{\partial L} = -c_i \frac{D}{Q} + \frac{1}{2} rC_P k \sigma L^{-\frac{1}{2}} \tag{7}$$

However, for fixed values of Q and θ , $TRC(Q, m, \theta, L)$ is concave in $L \in (L_i, L_{i-1})$, because

$$\frac{\partial^2 TRC(Q, m, \theta, L)}{\partial L^2} = -\frac{r}{4} C_P k \sigma L^{-\frac{3}{2}} < 0$$

Therefore, for fixed Q and θ , the minimum joint total expected annual cost will occur at the end-points of the interval. On the other hand, for a given value of $L \in [L_i, L_{i-1}]$, setting equations (5) and (6) equal to zero and solving for Q and θ , it follows that

$$Q = \left[\frac{2D(A + S/m + R(L))}{r(C_V(m(1 - D/P) - 1 + 2D/P) + C_P) + gmD\theta} \right]^{\frac{1}{2}} \quad L \in (L_i, L_{i-1}) \tag{8}$$

and

$$\theta = \frac{2iq}{gmDQ} \tag{9}$$

Theoretically, for fixed $L \in [L_i, L_{i-1}]$, one can find the optimal values of Q^* and θ^* from (8) and (9). In addition, for fixed $L \in [L_i, L_{i-1}]$, the Hessian matrix of $TRC(Q, m, \theta, L)$ is positive definite at Q^* and θ^* . The proof is shown in the appendix.

For a particular value of m , the total relevant annual cost is described by

$$TRC(m) = \left[2D \left(A + \frac{S}{m} + R(L) \right) \left(rC_V \left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) + rC_P + gmD\theta \right) \right]^{\frac{1}{2}} + rC_P k \sigma \sqrt{L} + iq \ln \frac{\theta_0}{\theta} \quad (10)$$

Ignore the terms that are independent of m , and take the square of (10); then, minimizing $TRC(m)$ is equivalent to minimizing

$$\begin{aligned} & (TRC(m))^2 \\ &= 2D \left(A + \frac{S}{m} + R(L) \right) \left(rC_V \left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) + rC_P + gmD\theta \right) \\ &= 2D \left[\left(A + R(L) \right) \left(rC_P - \left(1 - \frac{2D}{P} \right) rC_V \right) + S \left(rC_V \left(1 - \frac{D}{P} \right) + gD\theta \right) \right. \\ & \quad \left. + m \left(A + R(L) \right) \left(rC_V \left(1 - \frac{D}{P} \right) + gD\theta \right) + \frac{S}{m} \left(rC_P - \left(1 - \frac{2D}{P} \right) rC_V \right) \right] \end{aligned}$$

Once again, ignoring the terms that are independent of m , the minimization of the problem can be reduced to that of minimizing

$$Z(m) = m \left(A + R(L) \right) \left(rC_V \left(1 - \frac{D}{P} \right) + gD\theta \right) + \frac{S}{m} \left(rC_P - \left(1 - \frac{2D}{P} \right) rC_V \right) \quad (11)$$

The optimal value of $m = m^*$ is obtained when

$$Z(m^*) \leq Z(m^* - 1) \quad \text{and} \quad Z(m^*) \leq Z(m^* + 1). \quad (12)$$

Substituting relevant values in (12), the following condition holds:

$$m^*(m^* - 1) \leq \frac{S(rC_P - (1 - 2D/P)rC_V)}{(A + R(L))(rC_V(1 - D/P) + gD\theta)} \leq m^*(m^* + 1) \quad (13)$$

Hence, for fixed $L \in [L_i, L_{i-1}]$, when the constraint $0 < \theta \leq \theta_0$ is ignored, one can find the optimal values of Q^* , m^* and θ^* such that the annual total relevant cost reaches a minimum.

The following procedure is constructed to find optimal values of Q , m , θ and L for the problem under investigation.

Step 1. For each L_i , $i = 1, 2, \dots, n$, set $\theta_i = \theta_0$ and perform (i)–(iii):

- (i) Substitute θ_i into equation (13) to find m_i , and use θ_i and m_i to compute Q_i using equation (8).
- (ii) Use Q_i and m_i to determine θ_i from equation (9).
- (iii) Repeat (i)–(ii) until no change occurs in the values of Q_i , m_i and θ_i . Denote these solutions by Q_i^* , m_i^* and θ_i^* , respectively.

- Step 2. If $\theta_i^* \leq \theta_0$, then the solution found in step 1 is optimal for the given L_i ; so use equation (4) to compute $TRC(Q_i^*, m_i^*, \theta_i^*, L_i)$, for $i = 0, 1, \dots, n$, and go to step 4.
- Step 3. If $\theta_i^* > \theta_0$, set $\theta_i^* = \theta_0$ for the given L_i , then substitute θ_i^* into equation (13) to compute m_i^* , and use θ_i^* and m_i^* to determine Q_i^* from equation (8); so use equation (4) to calculate $TRC(Q_i^*, m_i^*, \theta_i^*, L_i)$, for $i = 0, 1, \dots, n$.
- Step 4. Set $TRC(Q_s, m_s, \theta_s, L_s) = \min_{i=0,1,\dots,n} \{TRC(Q_i^*, m_i^*, \theta_i^*, L_i)\}$. Then $TRC(Q_s, m_s, \theta_s, L_s)$ is a set of optimal solutions.

5. An illustrative example

Consider an inventory system with the following characteristics (Pan and Yang 2002): $D = 1000$ unit/year, $P = 3200$ unit/year, $A = \$25$ /order, $S = \$200$ /setup, $C_P = \$25$ /unit, $C_V = \$20$ /unit, $r = 0.2$, $k = 2.33$, $\sigma = 7$ unit/week, $i = 0.1$ \$/year, $g = \$15$ per defective unit, $q(\theta) = 400 \ln(0.0002/\theta)$, and the lead time has three components with data shown in table 1.

Applying the proposed solution procedure yields optimal lead time $L_s = 42$ days, optimal number of deliveries $m_s = 4$, optimal probability $\theta_s = 0.000010336$, and optimal order quantity $Q^* = 129$ units. The total relevant annual cost is \$2273.359. The resulting solution procedure is summarized in table 2.

Without crashing, frequent delivery and quality investment, the traditional integrated inventory model is given by:

$$TRC(Q) = \frac{D}{Q}(A + S) + \frac{Q}{2} \left(\frac{D}{P} r C_V + r C_P + g D \theta \right) + r C_P k \sigma \sqrt{L} \tag{14}$$

To obtain the minimum cost lot size, take the first derivative of $TRC(Q)$, and set it to zero; thus,

$$\frac{\partial TRC(Q)}{\partial Q} = -\frac{D}{Q^2}(A + S) + \frac{1}{2} \left(\frac{D}{P} r C_V + r C_P + g D \theta \right) = 0 \tag{15}$$

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

Table 1. Lead time data for the illustrative example.

i	L_i	m_i^*	Q_i^*	θ_i^*	$TRC(Q_i^*, m_i^*, \theta_i^*, L_i)$
0	56	4	129	0.000010336	2293.408
1	42	4	129	0.000010336	2273.359 ^a
2	28	3	170	0.000010458	2358.321
3	21	3	186	0.000009558	2532.913

^aThe minimum total relevant annual cost.

Table 2. Summary of the solution procedure for the illustrative example.

	Traditional model	Proposed model
Purchaser order lot size	303	129
Vender produce lot size	303	516
Lead time	56	42
Number of deliveries	1	4
Probability of process being out of control	0.0002	0.000010336
Joint total annual cost	3034.673	2273.359

Table 3. Summary of comparison for the illustrative example.

The function $TRC(Q)$ is convex in Q , since

$$\frac{\partial^2 TRC(Q)}{\partial Q^2} = \frac{2D}{Q^3}(A + S) > 0$$

Therefore, solving for the optimal order quantity in (15), we obtain

$$Q = \left[\frac{2D(A + S)}{r\left(\frac{D}{P}C_V + C_P\right) + gD\theta} \right]^{\frac{1}{2}} \quad (16)$$

From (16), the optimal order quantity is $Q^* = 303$ units and the corresponding minimum total relevant annual cost is \$3034.673. The summary of the comparison is presented in table 3. From table 3, the proposed model is shown to provide a lower total inventory cost with higher quality, more frequent delivery with smaller lot size and shorter lead time.

The philosophy of JIT purchasing is to establish a long-term relationship with vendors to maintain regulated shipments to minimize ordering cost and to buy enough parts as needed to avoid paying holding cost. However, these two costs will inevitably exist, though they may not be very significant, in many practical situations. The proposed integrated model can be shown to cope with these particular circumstances. For example, suppose that the fixed cost per order is as low as \$0.1 in the illustrative example. Applying the solution procedure it can be shown that the optimal lead time is $L_s^* = 56$ days, the optimal number of deliveries is $m_s^* = 70$, the optimal probability is $\theta_s^* = 0.000002067$, and the optimal order quantity Q^* is decreased to 8 units with total relevant annual cost of \$1903 and annual ordering cost of \$12.5. Compared to the original result, the holding cost is reduced because of smaller order quantity.

This result can also be obtained theoretically. From equation (13), the number of deliveries increases as the ordering cost decreases, which in turn causes the order quantity to decrease by equation (8). Consequently, the average inventory level and the associated annual holding cost are reduced. That is, an insignificant ordering cost in the integrated system will result in frequent delivery of small quantities with low on-hand inventory that lives up to the expectation of JIT philosophy. The JIT system emphasizes the reduction of inventory cost. There are several reasons why this is desirable, and the reduction of order quantity is certainly one of them.

6. Conclusions

Many researchers focus on the benefit from quality improvement or lead-time reduction in inventory models but only from a single party's viewpoint. However, consideration of the dyadic relationship between the vendor and purchaser is essential for implementing a just-in-time purchasing model. This paper investigates a JIT purchasing model where a single vendor supplies a single purchaser with a product by presenting an integrated inventory model that accounts for replenishment lead-time reduction and quality improvement investment considerations. The model is to minimize the sum of the ordering/setup cost, holding cost, quality improvement and crashing cost by simultaneously optimizing the order quantity, the lead time, the process quality and the number of deliveries with normally distributed lead time demand and is shown to provide a lower total cost, higher quality, smaller lot size and shorter lead time. JIT purchasing has an enormous impact on a company's profitability, especially in a competitive environment characterized by small profit margins. Furthermore, the application of JIT technologies such as small lot size, lead time reduction and quality improvement play a significant role in achieving JIT purchasing. The proposed model serves as a pioneering work on investigating the effects of lead time reduction and quality improvement on the integrated inventory model.

Appendix

For fixed $L \in (L_i, L_{i-1})$, $TRC(Q, m, \theta, L)$ is convex with respect to Q and θ , since

$$\frac{\partial^2 TRC(Q, m, \theta, L)}{\partial Q^2} = \frac{2D}{Q^3} \left(A + \frac{S}{m} + R(L) \right) > 0$$

$$\frac{\partial^2 TRC(Q, m, \theta, L)}{\partial \theta^2} = \frac{iq}{\theta^2} > 0$$

The determinant of the Hessian matrix can be calculated as

$$\begin{aligned} & \frac{\partial^2 TRC(Q, m, \theta, L)}{\partial Q^2} \cdot \frac{\partial^2 TRC(Q, m, \theta, L)}{\partial \theta^2} - \left[\frac{\partial^2 TRC(Q, m, \theta, L)}{\partial Q \partial \theta} \right]^2 \\ &= \frac{2D}{(Q^*)^3} \left(A + \frac{S}{m^*} + R(L) \right) \frac{iq}{(\theta^*)^2} - \left(\frac{gm^*D}{2} \right)^2 \\ &= \left(\frac{gm^*D}{2} \right)^2 \left[\frac{2D(A + \frac{S}{m^*} + R(L))}{iqQ^*} - 1 \right] \\ &= \left(\frac{gm^*D}{2} \right)^2 \left[\frac{Q^*(rC_V(m^*(1 - D/P) - 1 + 2D/P) + rC_P + gm^*D\theta^*)}{iq} - 1 \right] \\ &= \left(\frac{gm^*D}{2} \right)^2 \left[\frac{Q^*(rC_V(m^*(1 - D/P) - 1 + 2D/P) + rC_P)}{iq} + 1 \right] > 0 \end{aligned}$$

Hence, for a fixed L , the Hessian matrix is positive and $TRC(Q, m, \theta, L)$ is convex with respect to Q and θ .

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