

1. 解  $y'' - 3y' + 2y = e^{-4t}$ ,  $y(0) = 1$ ,  $y'(0) = 5$ .

<解>  $[s^2 Y(s) - sy(0) - y'(0)] - 3[sY(s) - y(0)] + 2Y(s) = \frac{1}{s+4}$

$$[s^2 Y(s) - s - 5] - 3[sY(s) - 1] + 2Y(s) = \frac{1}{s+4}$$

$$(s^2 - 3s + 2)Y(s) = s + 2 + \frac{1}{s+4}$$

$$Y(s) = \frac{s+2}{s^2 - 3s + 2} + \frac{1}{(s+4)(s^2 - 3s + 2)}$$

$$= \frac{s^2 + 6s + 9}{(s-1)(s-2)(s+4)}$$

$$= \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s+4}$$

$$= \frac{A(s-2)(s+4) + B(s-1)(s+4) + C(s-1)(s-2)}{(s-1)(s-2)(s+4)}$$

$$= \frac{s^2(A+B+C) + s(2A+3B-3C) + (-8A-4B+2C)}{(s-1)(s-2)(s+4)}$$

$$A+B+C=1 \quad (1)$$

$$2A+3B-3C=6 \quad (2)$$

$$-8A-4B+2C=9 \quad (3)$$

$$A = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 6 & 3 & -3 \\ 9 & -4 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -3 \\ -8 & -4 & 2 \end{vmatrix}} = \frac{-96}{30} = -\frac{16}{5}, \quad B = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 6 & -3 \\ -8 & 9 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -3 \\ -8 & -4 & 2 \end{vmatrix}} = \frac{125}{30} = \frac{25}{6}$$

$$C = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ -8 & -4 & 9 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -3 \\ -8 & -4 & 2 \end{vmatrix}} = \frac{1}{30}$$

$$Y(s) = -\frac{16}{5} \frac{1}{s-1} + \frac{25}{6} \frac{1}{s-2} + \frac{1}{30} \frac{1}{s+4}$$

$$y(t) = -\frac{16}{5} e^t + \frac{25}{6} e^{2t} + \frac{1}{30} e^{-4t}$$

