Differential form	Integral form	
$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$	$\oint_c \underline{E} \cdot \underline{dl} = -\frac{\partial}{\partial t}\int_s \underline{B} \cdot \underline{ds}$	Faraday's law
$\nabla \cdot \underline{D} = \rho$	$\int_s \underline{D} \cdot \underline{ds} = \int_v \rho dv$	Gauss's law
$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$	$\oint_c \underline{H} \cdot \underline{dl} = \int_s \underline{J} \cdot \underline{ds} + \frac{d}{dt}\int_s \underline{D} \cdot \underline{ds}$	Ampere's law
$\nabla \cdot \underline{B} = 0$	$\int_s \underline{B} \cdot \underline{ds} = 0$	
$\underline{D} = \varepsilon \underline{E}$		Constitutive relations
$\underline{H} = \frac{\underline{B}}{\mu}$		
$\nabla \cdot \underline{J} = -\frac{\partial \rho}{\partial t}$	$\int_s \underline{J} \cdot \underline{ds} = -\frac{\partial}{\partial t}\int_v \rho dv$	Equation of continuity

$$\nabla \cdot \underline{B} = 0, \qquad \therefore \underline{B} = \nabla \times \underline{A}$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}, \qquad \nabla \times \left(\underline{E} + \frac{\partial \underline{A}}{\partial t}\right) = 0, \qquad \therefore \underline{E} + \frac{\partial \underline{A}}{\partial t} = -\nabla \varphi$$

$$\underline{E} = -\nabla \varphi - \frac{\partial \underline{A}}{\partial t}$$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}, \qquad \nabla \times \mu \underline{H} = \mu \underline{J} + \mu\varepsilon \frac{\partial \underline{E}}{\partial t}$$

$$\nabla \times \underline{B} = \mu \underline{J} + \mu\varepsilon \frac{\partial \underline{E}}{\partial t}$$

$$\nabla \times \nabla \times \underline{A} = \mu \underline{J} + \mu\varepsilon \frac{\partial}{\partial t}\left(-\nabla \varphi - \frac{\partial \underline{A}}{\partial t}\right)$$

$$\nabla\left(\nabla \cdot \underline{A}\right) - \nabla^2 \underline{A} = -\mu\varepsilon \nabla \frac{\partial \varphi}{\partial t} - \mu\varepsilon \frac{\partial^2 \underline{A}}{\partial t^2}$$

$$\nabla^2 \underline{A} - \mu\varepsilon \frac{\partial^2 \underline{A}}{\partial t^2} - \nabla\left(\nabla \cdot \underline{A} + \mu\varepsilon \frac{\partial \varphi}{\partial t}\right) = -\mu \underline{J}, \qquad \nabla \cdot \underline{A} + \mu\varepsilon \frac{\partial \varphi}{\partial t} = 0$$ Lorentz gauge

$$\nabla^2 \underline{A} - \mu\varepsilon \frac{\partial^2 \underline{A}}{\partial t^2} = -\mu \underline{J}$$ nonhomogeneous wave equation for vector potential $\underline{A}$

$$\nabla \cdot \underline{D} = \rho, \qquad \nabla \cdot \underline{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \cdot \left(-\nabla \varphi - \frac{\partial \underline{A}}{\partial t} \right) = \frac{\rho}{\varepsilon}$$

$$\nabla^2 \varphi + \nabla \cdot \frac{\partial \underline{A}}{\partial t} = -\frac{\rho}{\varepsilon}, \qquad \nabla \cdot \underline{A} = -\mu\varepsilon \frac{\partial \varphi}{\partial t}, \qquad \nabla \cdot \frac{\partial \underline{A}}{\partial t} = -\mu\varepsilon \frac{\partial^2 \varphi}{\partial t^2}$$

$$\nabla^2 \boldsymbol{\varphi} - \boldsymbol{\mu\varepsilon} \frac{\partial^2 \boldsymbol{\varphi}}{\partial \boldsymbol{t}^2} = -\frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon}}$$ nonhomogeneous wave equation for scalar potential $\boldsymbol{\varphi}$

$$\underline{A}(\underline{r}, t) = \frac{\mu}{4\pi} \int_v \frac{\underline{J}\left(\underline{r}', t - \frac{|\underline{r} - \underline{r}'|}{c} \right)}{|\underline{r} - \underline{r}'|} dv'$$

$$\varphi(\underline{r}, t) = \frac{\mu}{4\pi} \int_v \frac{\rho\left(\underline{r}', t - \frac{|\underline{r} - \underline{r}'|}{c} \right)}{|\underline{r} - \underline{r}'|} dv$$

Waves equations in a source-free region

$\boldsymbol{\rho = 0}, \quad \underline{\boldsymbol{J}} = \boldsymbol{0}$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \nabla \times \underline{E} = -\mu \frac{\partial}{\partial t}\left(\nabla \times \underline{H} \right)$$

$$\nabla\left(\nabla \cdot \underline{E} \right) - \nabla^2 \underline{E} = -\mu \frac{\partial}{\partial t}\left(\frac{\partial \underline{D}}{\partial t} \right)$$

$$\nabla^2 \underline{E} - \mu\varepsilon \frac{\partial^2 \underline{E}}{\partial t^2} = 0$$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$$

$$\nabla \times \nabla \times \underline{H} = \varepsilon \frac{\partial}{\partial t} \nabla \times \underline{E}$$

$$\nabla\left(\nabla \cdot \underline{H} \right) - \nabla^2 \underline{H} = \varepsilon \frac{\partial}{\partial t}\left(-\frac{\partial \underline{B}}{\partial t} \right)$$

$$\nabla^2 \underline{H} - \mu\varepsilon \frac{\partial^2 \underline{H}}{\partial t^2} = 0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]E_x-\mu\varepsilon\frac{\partial^2 E_x}{\partial t^2}=0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]E_y-\mu\varepsilon\frac{\partial^2 E_y}{\partial t^2}=0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]E_z-\mu\varepsilon\frac{\partial^2 E_z}{\partial t^2}=0$$

Uniform plane wave is a particular solution of Maxwell's equations with **E** assuming the same direction, same magnitude, and same phase in infinite planes perpendicular to the direction of propagation (similarly for **B**).

uniform plane waves ***E*** propagating in the z direction

assuming $\frac{\partial \boldsymbol{E}}{\partial x}=\frac{\partial \boldsymbol{E}}{\partial y}=\mathbf{0}, \quad \frac{\partial \boldsymbol{E}}{\partial z}\neq \mathbf{0}$

$$\frac{\partial^2 \underline{\underline{E}}}{\partial z^2}-\mu\varepsilon\frac{\partial^2 \underline{\underline{E}}}{\partial t^2}=0$$

$$\frac{\partial^2 E_x}{\partial z^2}-\mu\varepsilon\frac{\partial^2 E_x}{\partial t^2}=0$$

$$\frac{\partial^2 E_y}{\partial z^2}-\mu\varepsilon\frac{\partial^2 E_y}{\partial t^2}=0$$

$$\frac{\partial^2 E_z}{\partial z^2}-\mu\varepsilon\frac{\partial^2 E_z}{\partial t^2}=0$$

phase velocity $\boldsymbol{v_p=1/\sqrt{\mu\varepsilon}}$, in free space, $\boldsymbol{v_p=1/\sqrt{\mu_0\varepsilon_0}\cong 3\times 10^8\, m/s}$

Assuming only $\boldsymbol{E_x}$ exists

$$\Rightarrow \quad \frac{\partial^2 \boldsymbol{E_x}}{\partial z^2}-\frac{\mathbf{1}}{\boldsymbol{v_p}^2}\frac{\partial^2 \boldsymbol{E_x}}{\partial t^2}=\mathbf{0}$$

Its general solution is of the form $\boldsymbol{E_x(z,t)=p_1(z-v_p t)+p_2(z+v_p t)}$

$$Let \qquad \zeta=z-v_p t,\quad \zeta=z+v_p t \qquad \Rightarrow \qquad E_x(z,t)=p_1(\zeta)+p_2(\zeta)$$

$$\frac{\partial E_x}{\partial z}=\frac{\partial E_x}{\partial \zeta}\frac{\partial \zeta}{\partial z}+\frac{\partial E_x}{\partial \eta}\frac{\partial \eta}{\partial z}=\frac{\partial E_x}{\partial \zeta}+\frac{\partial E_x}{\partial \eta};$$

$$\frac{\partial^2 E_x}{\partial z^2}=\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial z}\right)=\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial \zeta}+\frac{\partial E_x}{\partial \eta}\right)$$

$$=\frac{\partial}{\partial \zeta}\left(\frac{\partial E_x}{\partial \zeta}+\frac{\partial E_x}{\partial \eta}\right)\frac{\partial \zeta}{\partial z}+\frac{\partial}{\partial \eta}\left(\frac{\partial E_x}{\partial \zeta}+\frac{\partial E_x}{\partial \eta}\right)\frac{\partial \eta}{\partial z}$$

$$=\frac{\partial^2 E_x}{\partial \zeta^2}+2\frac{\partial^2 E_x}{\partial \zeta\partial \eta}+\frac{\partial^2 E_x}{\partial \eta^2}$$

likewise, since $\partial\zeta / \partial t = -v_p$ and $\partial\eta / \partial t = v_p$

$$\frac{\partial E_x}{\partial t} = -v_p \frac{\partial E_x}{\partial \zeta} + v_p \frac{\partial E_x}{\partial \eta}$$

$$\frac{\partial^2 E_x}{\partial t^2} = -v_p \frac{\partial}{\partial \zeta}\left(-v_p \frac{\partial E_x}{\partial \zeta} + v_p \frac{\partial E_x}{\partial \eta}\right) + v_p \frac{\partial}{\partial \eta}\left(-v_p \frac{\partial E_x}{\partial \zeta} + v_p \frac{\partial E_x}{\partial \eta}\right)$$

$$= v_p^{\ 2}\left(\frac{\partial^2 E_x}{\partial \zeta^2} - 2\frac{\partial^2 E_x}{\partial \zeta \partial \eta} + \frac{\partial^2 E_x}{\partial \eta^2}\right)$$

$$\therefore \quad \frac{1}{v_p^{\ 2}}\frac{\partial^2 E_x}{\partial t^2} = \frac{\partial^2 E_x}{\partial \zeta^2} - 2\frac{\partial^2 E_x}{\partial \zeta \partial \eta} + \frac{\partial^2 E_x}{\partial \eta^2}$$

$$\frac{\partial^2 E_x}{\partial \zeta^2} + 2\frac{\partial^2 E_x}{\partial \zeta \partial \eta} + \frac{\partial^2 E_x}{\partial \eta^2} = \frac{\partial^2 E_x}{\partial \zeta^2} - 2\frac{\partial^2 E_x}{\partial \zeta \partial \eta} + \frac{\partial^2 E_x}{\partial \eta^2}$$

$$\therefore \frac{\partial^2 E_x}{\partial \zeta \partial \eta} = 0$$

$E_x(z,t) = p_1(z - v_p t) + p_2(z + v_p t)$ general solution

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

$$E_x = X(z)T(t)$$

$$T\frac{\partial^2 X}{\partial z^2} - \mu\varepsilon X \frac{\partial^2 T}{\partial t^2} = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial z^2} - \underbrace{\mu\varepsilon \frac{1}{T}\frac{\partial^2 T}{\partial t^2}}_{=-k^2} = 0$$

$$\mu\varepsilon \frac{\partial^2 T}{\partial t^2} = -k^2 T \quad \Rightarrow \quad \frac{\partial^2 T}{\partial t^2} = -\frac{k^2}{\mu\varepsilon} T = -(kv_p)^2 T = -\omega^2 T$$

$$T = C_1 \cos\omega t + C_2 \sin\omega t$$

$$\frac{\partial^2 X}{\partial z^2} + k^2 X = 0$$

$$X = D_1 \cos kz + D_2 \sin kz$$

Assuming $C_2 = D_2 = 0$

$$E_x = X(z)T(t) = \frac{C_1 D_1}{2}\left[\cos(\omega t - kz) + \cos(\omega t + kz)\right]$$