Lecture 6
Solar Cells

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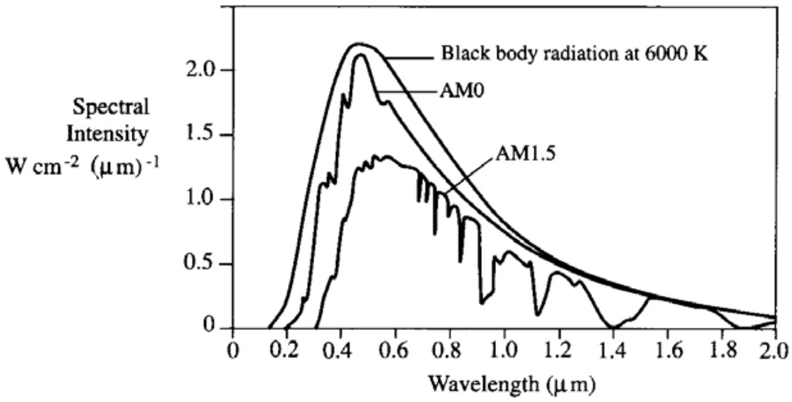
- Solar Energy Spectrum
- Optical Absorption
- Solar Cells Principles
- pn Junction Photovoltaic I-V Characteristics
- Series Resistance and Equivalent Circuit
- Temperature Effects

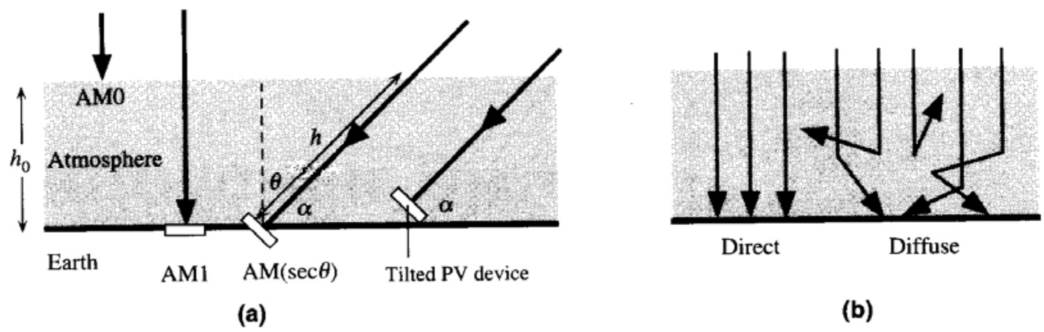
1. Solar Energy Spectrum

The actual intensity spectrum on Earth's surface depends on the absorption and scattering effects of the atmosphere and hence on the atmospheric composition and the radiation path length through the atmosphere.



Integration of spectral intensity over the whole spectrum gives the integrated or total intensity *I*.
The integrated intensity above Earth's atmosphere gives total power flow through a unit area perpendicular to the direction of the sun. This quantity is called the **solar constant** or **air-mass zero** (AM0) radiation and it is approximately constant at a value of 1353 Wm^{-2}.

■ Air mass *m* (AM*m*)



-Air mass *m* (AM*m*) is defined as the ratio of the actual radiation path *h* to the shortest path h_0, that is $m = h/ h_0$.
Since $h = h_0\sec\theta$, AM*m* is AMsecθ.
- AM0: I = 1353 Wm^{-2}.
AM1: θ = 0°, I = 925 Wm^{-2}.
AM1.5: θ = 48.2°, I = 844 Wm^{-2}.

■ Diffuse component

Scattering makes the sun's rays scattered in many directions. Consequently, the terrestrial light has a diffuse component in addition to the direct component.

On a clear day, the diffuse component can be roughly 20% of the total radiation, and significantly higher on cloudy days.

Ex. Solar energy conversion

Suppose that a particular family house in a sunny geographic location over a year consumes a daily average electrical power of 500 W. If the annual solar intensity incident per day is about 6 kW h m^{-2}, and a solar cell that converts solar energy to electrical energy has an efficiency of 15%, what is the required device area?

Sol:

Average energy consumed in one day = $500\times24\times60\times60 = 4.32\times10^7$ J.

Total energy available per unit area in one day = $6000\times60\times60\times0.15 = 0.324\times10^7$ J.

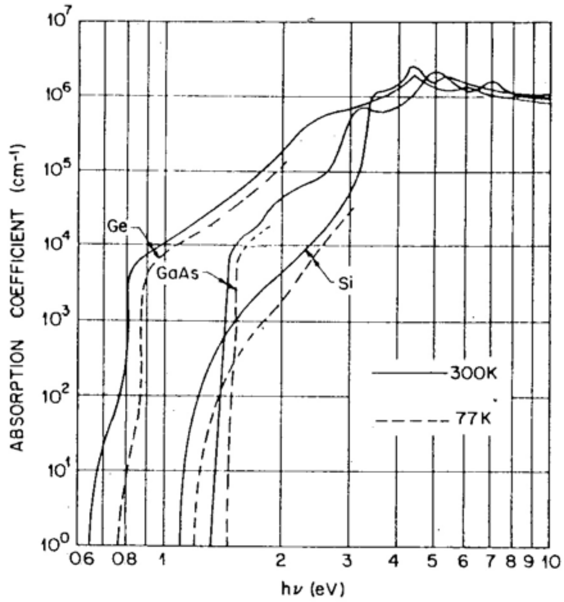
Required device area = 4.32/0.324 = 13.3 m^2.

2. Optical Absorption

The amount of absorption is expressed by using absorption coefficient described by Lambert's law, as

$$I = I_0 \exp(-\alpha x),$$

where I is light intensity, I_0 is the initial intensity, x is the distance, and α is the absorption coefficient (in cm^{-1}).
The inverse of the absorption coefficient is called the absorption length (or the penetration length). At the absorption length, the intensity has decreased to 1/e (36.8%).



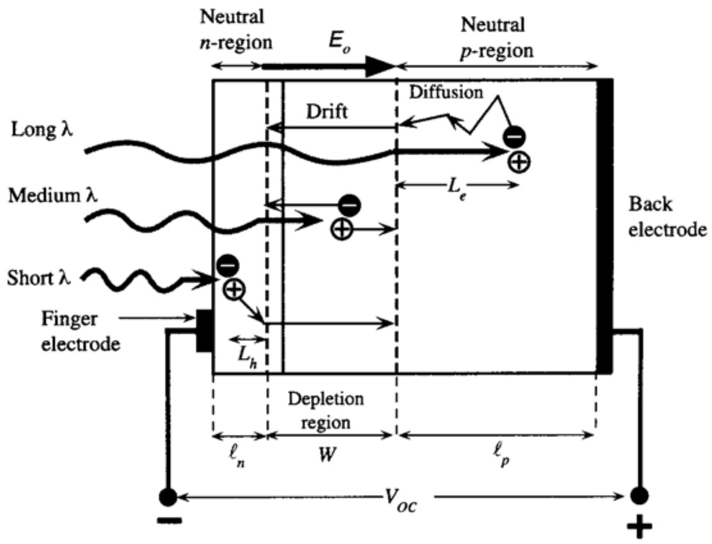
Photon energy (eV)	1	2	3	4	5
Wavelength	1.24 µm	621 nm	414 nm	311 nm	248 nm

3. Solar Cells Principles

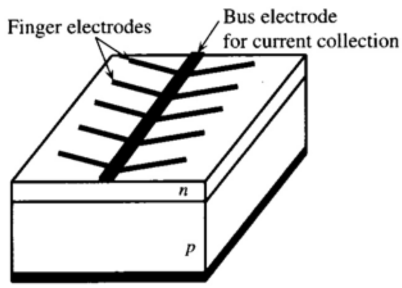
- Consider a pn junction with a very narrow and more heavily doped n-region. The illumination is through the thin n-side.

- Finger electrodes are deposited on the surface of n-side. A thin antireflection coating on the surface reduces reflections.

■ **Principle of operation**



The principle of operation of the solar cell.



Finger electrodes.

- The depletion region (W) or the space charge layer (SCL) extends primarily into the p-side because of more heavily doped n-region. There is a built-in field E_0 in this depletion region.

- As the n-side is very narrow, most of the photons are absorbed within the depletion region (W) and within the neutral p-side (l_p) and photogenerate electron-hole pairs (EHPs) in these regions.

• In depletion region
- EHPs photogenerated in the depletion region are immediately separated by the built-in field E_0 which drifts them apart.

- An open circuit voltage develops between the terminals of the device with the p-side positive and n-side negative.

- If an external load is connected then the excess electron in the n-side can travel around the external circuit, do work, and reach the p-side to recombine with the excess hole there.

• In neutral region

- The EHPs photogenerated by long wavelength photons that are absorbed in the neutral p-side can only diffuse in this region as there is no electric field.

- Those electrons within a distance L_e to the depletion region can readily diffuse and reach this region whereupon they become drifted by E_0 to the n-side. Holes left behind in the p-side contribute a net positive charge to this region.

$$L_e = \sqrt{D_e \tau_e}$$

L_e: electron diffusion length in the p-region (mean diffuse distance before recombination)
τ_e: recombination lifetime of the electron in the p-region
D_e: diffuse coefficient of the electron

It is important to have <u>minority carrier diffusion length L_e as long as possible</u>.

This is the reason that p-side is thicker than n-side; the electron diffusion length in Si is longer than the hole diffusion length.

- The same ideas also apply to EHPs photogenerated by short-wavelength photons absorbed in the n-side. Those holes photogenerated within a diffusion length L_h can reach the depletion region and become swept across the p-side.

• **photocurrent**
- If the terminals of the device are shorted, then the excess electron in the n-side can flow through the external circuit to neutralize the excess hole in the p-side. This current due to the flow of photogenerated carriers is called the **photocurrent**.

It is important to realize that under a steady state operation, there should be no net current through an open circuit solar cell. This means that the photocurrent inside the device due to the flow of phtogenerated carriers must be exactly balanced by a flow of carriers in the opposite directions. The latter carriers are minority carriers injected by the photovoltaic voltage across the *pn* junction as in a normal diode.

Since is L_h small, the n-side is made very thin, typically less than 0.2μm. Since L_e is larger than L_h and the absorption depth for long wavelengths (about 1μm) is greater than 100 μm, the p-side is 180-250 μm.

• Remarks about efficiency

Crystalline silicon has a bandgap of 1.1 eV which corresponds to a threshold wavelength of 1.1 μm.

(1) The incident energy in the wavelength region greater than 1.1 μm is then waste and it accounts for ~25%.

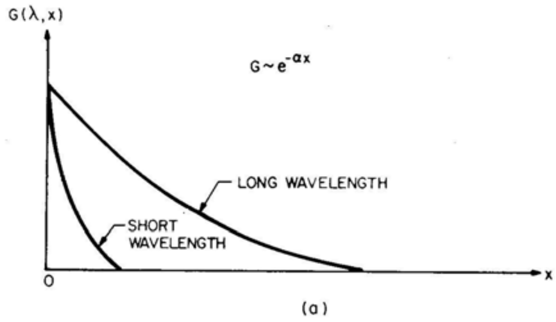
(2) The worst part of efficiency limitation comes from surface recombination in the n-side. This loss can be as high as 40%.

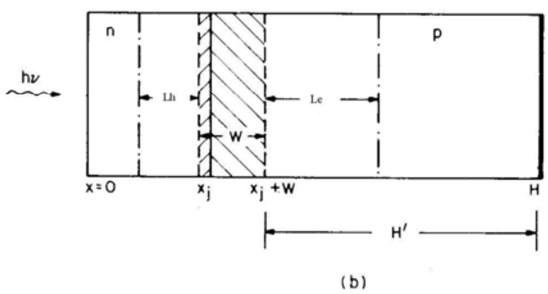
(3) The AR coating is not perfect which reduces the total collected photons by a factor of about 0.8-0.9.

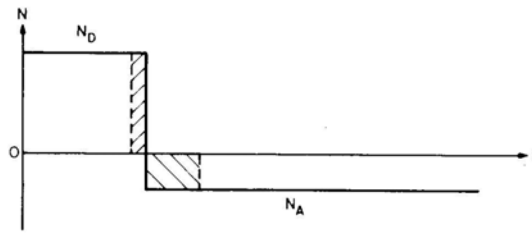
4. pn Junction Photovoltaic I-V Characteristics

■ **Photocurrent** [Sze]

● General case







Assume a monochromatic light of wavelength λ is incident on the front surface of a solar cell. The generation rate EHPs at a distance *x* from the semiconductor surface is given by

$G(\lambda, x) = G_0(\lambda)\exp[-\alpha(\lambda)x]$,

$G_0(\lambda) = \alpha(\lambda)\, F(\lambda)[1-R(\lambda)]$, (光一進入 Si 時，每秒產生的 EHPs)

where α(λ) is the absorption coefficient, *F*(λ) is the number of incident photons/cm^2/per unit bandwidth, and *R*(λ) is the reflectance of the surface.

Under low-injection condition (when the injection carrier density is much less than the equilibrium majority carrier density), we can solve steady-state continuity equations for electron density in p-type semiconductors and hole density in n-type semiconductors (steady-state: $\frac{\partial p_n}{\partial t} = \frac{\partial n_p}{\partial t} = 0$).

Assume free carriers from the depletion region are not considered at this moment.

(a) Hole photocurrent density at the depletion edge $x = x_j$

In the n-side the resulting hole photocurrent density at the depletion edge, $x = x_j$, is

$$J_p = -qD_h(\frac{dp_n}{dx})|_{x_j} = [qG_0L_h/(\alpha^2L_h^2-1)] \times [\frac{(\frac{S_hL_h}{D_h}+\alpha L_h) - e^{-\alpha x_j}(\frac{S_hL_h}{D_h}\cosh\frac{x_j}{L_h}+\sinh\frac{x_j}{L_h})}{(S_hL_h/D_h)\sin(x_j/L_h)+\cosh(x_j/L_h)} - \alpha L_h e^{-\alpha x_j}], \quad (1)$$

where S_h is surface recombination velocity at $x = 0$ and $L_h = (D_h\tau_h)^{1/2}$ is the diffusion length of holes.

(b) Electron photocurrent density at the depletion edge $x = x_j+W$

In the p-side the resulting electron photocurrent density at the depletion edge, $x = x_j+W$, is

$$J_n = qD_e(\frac{dn_p}{dx})|_{x_j+W} = [qG_0L_e/(\alpha^2L_e^2-1)]\exp[-\alpha(x_j+w)] \times \{\alpha L_e - \frac{\frac{S_eL_e}{D_e}[\cosh(H'/L_e) - \exp(-\alpha H')] + \sinh(H'/L_e) + \alpha L_e \exp(-\alpha H')}{(S_eL_e/D_e)\sinh(H'/L_e)+\cosh(H'/L_e)}\} \quad (2)$$

where S_e is surface recombination velocity at $x = H$ and L_e is the diffusion length of electrons.

(c) Photocurrent density

Some photocurrent generation takes place within the depletion region as well. The photogenerated carriers are accelerated out of the depletion region before they can recombine. The photocurrent density is

$$J_{dr} = qF(1-R)\exp(-\alpha x_j)[1-\exp(-\alpha W)]. \quad (3)$$

Therefore, the total photocurrent density at a given length is

$$J_{ph}(\lambda) = J_p(\lambda) + J_n(\lambda) + J_{dr}(\lambda). \quad (4)$$

The spectral response (SR) is equal to this sum divided by qF(1-R) for internal spectral response:

$$SR(\lambda) = \frac{J_p(\lambda) + J_n(\lambda) + J_{dr}(\lambda)}{qF(\lambda)[1 - R(\lambda)]}. \tag{5}$$

Once the spectral response is known, the total photocurrent density is given by

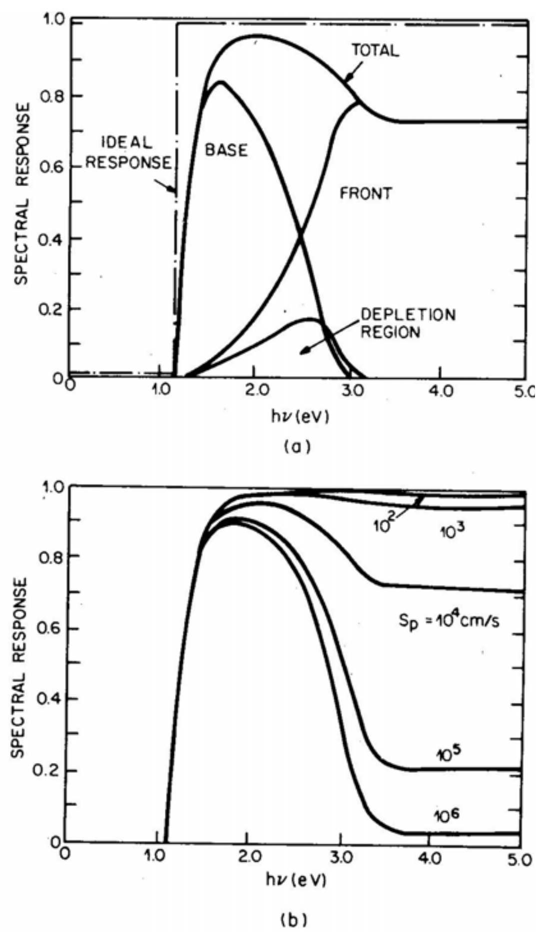
$$J = q\int_0^{\lambda_m} F(\lambda)[1 - R(\lambda)]SR(\lambda)d\lambda \tag{6}$$

where λ_m is the longest wavelength corresponding to the semiconductor bandgap.

To obtain large *J*, one should

(a) minimize $R(\lambda)$,

(b) maximize $SR(\lambda)$: increase L_e and L_h, and decrease S_e and S_h.



Parameter	Value
A	100 cm^2
W_N	0.35 μm
N_D	1×10^{20} cm^{-3}
D_p	1.5 cm^2/V-s
$S_{F.eff}$	3×10^4 cm/s
τ_p	1 μs
L_p	12 μm
W_P	300 μm
N_A	1×10^{15} cm^{-3}
D_n	35 cm^2/V-s
S_{BSF}	100 cm/s
τ_n	350 μs
L_n	1100 μm

Typical values for parameters

- Simplified case [Kasap]

Consider a photovoltaic device that is illuminated by a monochromatic light. Suppose that L_h is greater than the n-layer thickness l_n so that all the EHPs generated within the volume (l_n+W+L_e) contribute to the photocurrent. Further, assume that EHP recombination near the crystal surface is negligible.

The EHP photogeneration rate follows

$$G_{ph} = G_0 \exp(-\alpha x).$$

Total number of EHP generated per unit time in l_n+W+L_e is

$$dN_{EHP}/dt = A\int_0^{l_n+W+L_e} G_0 \exp(-\alpha x)dx,$$

where N_{EHP} is total number of EHP generated within the volume (l_n+W+L_e).

$$\frac{dN_{EHP}}{dt} = \frac{G_0 A}{\alpha}\{1-\exp[-\alpha(l_n+W+L_e)]\}$$

Since the photogenerated electrons flow through the external circuit, the photocurrent I_{ph} is then $q(dN_{EHP}/dt)$.

$$I_{ph} = \frac{qG_0 A}{\alpha}\{1-\exp[-\alpha(l_n+W+L_e)]\} \quad (7)$$

For long wavelengths, α will be small. Expanding the exponential by

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \approx 1 + x,$$

we find

$$I_{ph} = qG_0 A(l_n+W+L_e) \quad (8)$$

Ex. Taking a crystalline Si solar cell that has A=5cm×5cm, l_n =0.5μm, W=2 μm, and L_e =50 μm. Assume G_0=10^{18}cm^{-3}s^{-1}at λ≈1.1 μm and G_0=10^{19}cm^{-3}s^{-1}at λ≈0.83 μm.

(a) At λ≈1.1 μm: α=2000m^{-1}(absorption depth = 1/α = 500 μm)

$$I_{ph} = \frac{1.6\times10^{-19}\times10^{18}\times10^{6}\times25\times10^{-4}}{2000}\{1-\exp[-2000(52.5\times10^{-6})]\}$$

I_{ph} = 20 mA

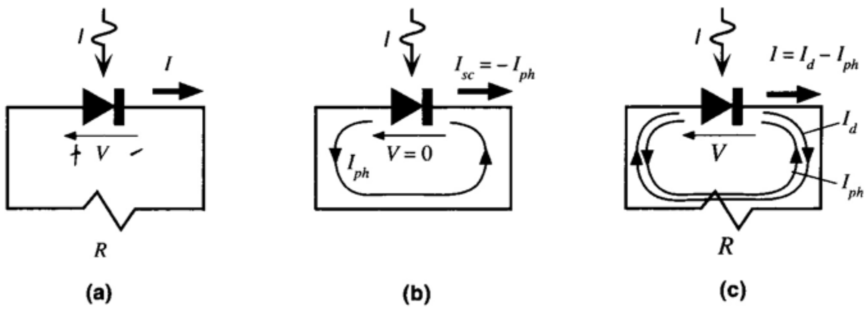
(b) At λ≈0.83 μm: α=10^5m^{-1}(absorption depth = 1/α = 10 μm)

$$I_{ph} = \frac{1.6\times10^{-19}\times10^{19}\times10^{6}\times25\times10^{-4}}{10^5}\{1-\exp[-10^5(52.5\times10^{-6})]\}$$

I_{ph} = 40 mA

■ I-V Characteristics

Consider an ideal *pn* junction photovoltaic device connected to a resistive load *R*.



If the load is a short circuit, then the only current in the circuit is that generated by the incident light. This short circuit current is I_{sc} = -I_{ph} = -*KI*, where *I* is the light intensity and *K* is a constant that depends on the particular device.

If R is not a short circuit, then a positive voltage *V* appears across the *pn* junction as a result of the current passing through it. Thus, in addition to I_{ph} there is also a forward diode current I_d in the circuit just as it would in a normal diode. The forward diode current I_d is given by the diode characteristics,

$$I_d = I_0[\exp(\frac{qV}{nk_BT})-1], \quad (9)$$

where I_0 is the reverse saturation current and n is the ideality factor that depends on the semiconductor material and fabrication characteristics (n = 1~2).

The total current through the solar cell is

$$I = -I_{ph} + I_0[\exp(\frac{qV}{nk_BT}) - 1] \quad (10)$$

- Open circuit voltage

For open circuit, we have

$$I = -I_{ph} + I_0[\exp(\frac{qV_{oc}}{nk_BT}) - 1] = 0$$

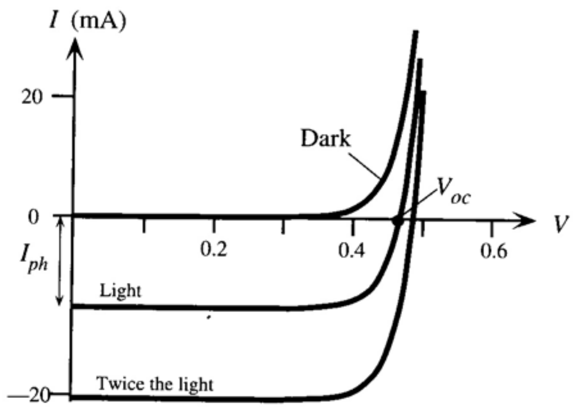
Assuming the open circuit voltage $V_{oc} >> nk_BT/q$, rearranging the above equation we can obtain

$$V_{oc} = \frac{nk_BT}{q}\ln(\frac{I_{ph}}{I_0}) \quad (11)$$

-V_{oc} depends on light intensity.
-V_{oc} typically lies in the range 0.4-0.6V.

- I-V Characteristics

The overall I-V Characteristics of a typically Si solar cell is shown below.

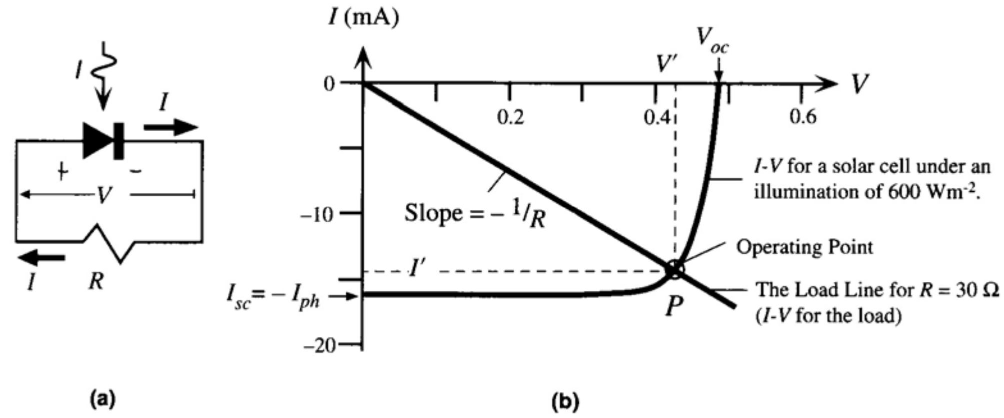


When the solar cell is connected to a resistive load R, we have

$$I = -\frac{V}{R} \quad (12)$$

The actual current and voltage in the circuit can be obtained by solving Eqs. (11)

and (12) simultaneously, but this is not a trivial analytical procedure. A graphical solution using the solar cell characteristics however is straightforward.



• Fill Factor

The fill factor, which is a figure of merit for the solar cell, is defined as

$$FF = \frac{I_m V_m}{I_{sc} V_{oc}}, \tag{13}$$

where $I_m V_m$ is the maximum power delivered to the load, I_m and V_m are the corresponding operating current and voltage, and I_{sc} and V_{oc} are short circuit current and open circuit voltage. Since the maximum possible current is I_{sc} and the maximum possible voltage is V_{oc}, I_{sc} V_{oc} represents the desirable goal in power delivery for a given solar cell.

(a) FF is a measure of the closeness of the solar cell *I*-*V* curve to the rectangular shape (the ideal shape).
(b) It is advantageous to have FF as close to unity as possible.
(c) Typically FF values are in the range of 70-85% and depend on the device material and structure.

• Conversion Efficiency

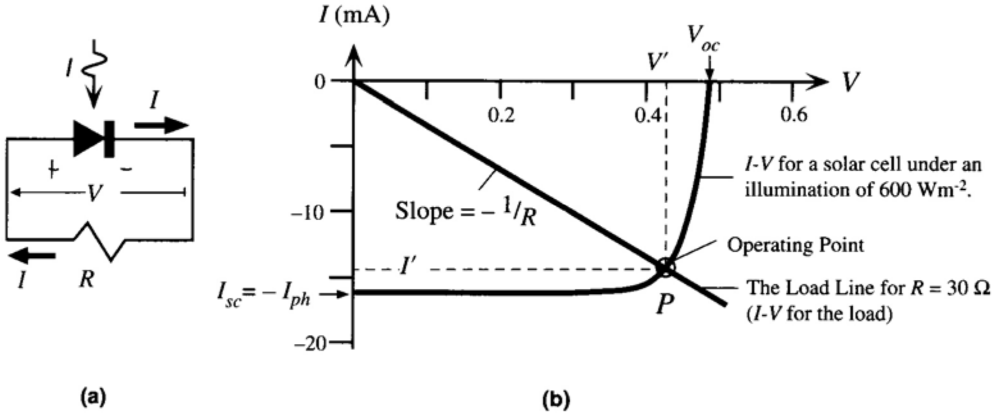
The conversion efficiency is defined as

$$\eta = \frac{I_m V_m}{P_{in}} = \frac{FF \bullet I_{sc} V_{oc}}{P_{in}}, \tag{14}$$

where P_{in} is the optical power incident on the solar cell.

Ex. A solar cell driving a resistive load

Consider a solar cell driving a 30Ω resistive load. Suppose that the cell has an area of 1cm×1cm and is illuminated with light of intensity 600Wm^{-2}. What are the current and voltage in the circuit? What is the power delivered to the load? What is the efficiency of the solar cell in this circuit?



Sol:

(a) From figure (b), $I' = 14.2mA$ and $V' = 0.425V$.

(b) $P_{out} = I'V' = 14.2 \times 0.425 = 6.035mW$

(c) P_{in} = (light intensity)(surface area) = (600 Wm^{-2})(0.01m)^2 = 60mW

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{6.035}{60} \times 100\% = 10.06\%$$

Note that this efficiency is not necessarily the maximum efficiency available from this solar cell.

Ex. Open circuit voltage and illumination

A solar cell under an illumination of 600 Wm^{-2} has a short circuit current I_{sc} of 16.1 mA and an open circuit voltage V_{oc} of 0.485V. What are the short circuit current and open circuit voltage when the intensity is double?

Sol:

Assuming the open circuit voltage $V_{oc} >> nk_BT/q$, we can obtain

$$V_{oc} = \frac{nk_BT}{q}\ln(\frac{I_{ph}}{I_0})$$

In the above equation, the photocurrent I_{ph} depends on the light intensity I via $I_{ph}=KI$.

At a given temperature, the change in V_{oc} is

$$V_{oc2} - V_{oc1} = \frac{nk_BT}{q}[\ln(\frac{I_{ph2}}{I_0}) - \ln(\frac{I_{ph1}}{I_0}) = \frac{nk_BT}{q}[(\ln I_{ph2} - \ln I_0) - (\ln I_{ph1} - \ln I_0)]$$

$$= \frac{nk_BT}{q}(\ln I_{ph2} - \ln I_{ph1}) = \frac{nk_BT}{q}\ln(\frac{I_{ph2}}{I_{ph1}})$$

$$V_{oc2} - V_{oc1} = \frac{nk_BT}{q}\ln(\frac{I_{ph2}}{I_{ph1}}) = \frac{nk_BT}{q}\ln(\frac{I_2}{I_1})$$

(a) The short circuit current for the double intensity is

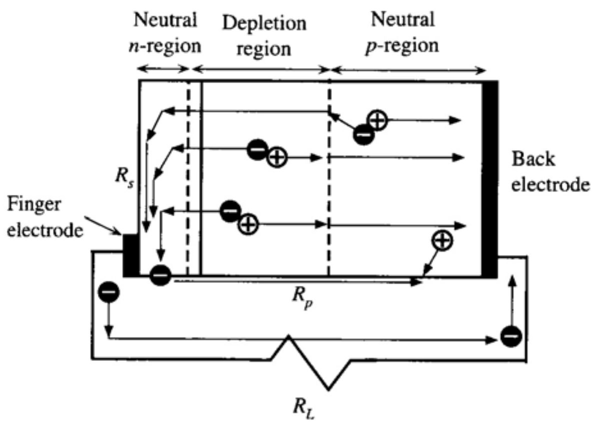
$$I_{sc2} = I_{sc1}(\frac{I_2}{I_1}) = 16.1 \times 2 = 32.2mA$$

(b) Assuming $n = 1$, the new open circuit voltage is

$$V_{oc2} = V_{oc1} + \frac{nk_BT}{q}\ln(\frac{I_2}{I_1}) = 0.485 + 1(0.0259)\ln(2) = 0.503V$$

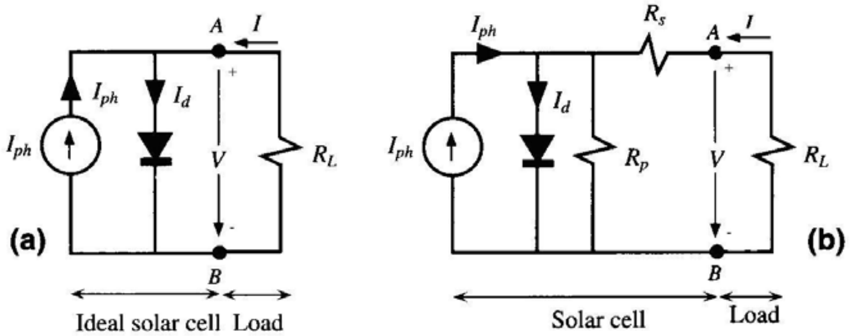
■ Series Resistance and Equivalent Circuit

All these electron paths in the n-layer surface region to finger electrodes introduce an effective series resistance R_s into the photovoltaic circuit. There is also a series resistance due to the neutral p-region but this generally small compared with the resistance of the electron paths to the finger electrodes.



The figure shows the equivalent circuit of a more practical solar cell. A fraction (usually small) of the phtogenerated carriers can also flow through the crystal surfaces (edges of the devices) or through grain boundaries in polycrystalline devices instead of flowing through the external load R_L. These effects can be

represented by an effective internal shunt or parallel resistance R_p that diverts the photocurrent away from the load R_L.
Typically R_p is less important than R_s in overall device behavior.

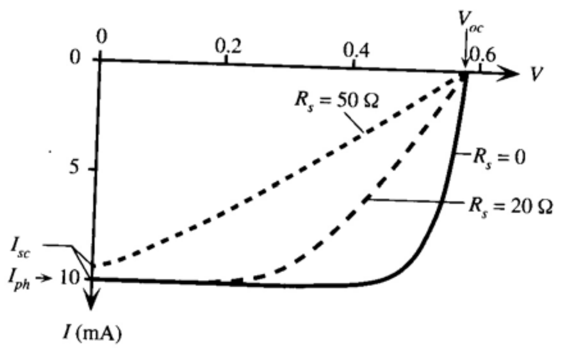


$$I = -I_{ph} + \frac{V - IR_s}{R_p} + I_o[\exp(\frac{q}{nk_BT}(V - IR_s)) - 1]$$

$$I(1 + \frac{R_s}{R_p}) = -I_{ph} + \frac{V}{R_p} + I_o[\exp(\frac{q}{nk_BT}(V - IR_s)) - 1]$$

$$I = (\frac{R_p}{R_s + R_p})\{-I_{ph} + I_o[\exp(\frac{q}{nk_BT}(V - IR_s)) - 1]\} + \frac{V}{R_s + R_p}$$

<u>The series resistance R_s can significantly deteriorate the solar cell performance</u>.



■ Temperature Effects

<u>The output voltage and efficiency of a solar cell increases with decreasing temperature</u>.

The output voltage, V_{oc}, when $V_{oc} >> nk_BT/q$ is given by,

$$V_{oc} = \frac{nk_BT}{q}\ln(\frac{I_{ph}}{I_0}) \quad (11)$$

Since $I_{ph} = KI$, where K is a constant and I is the light intensity, Eq. (11) can be written as

$$V_{oc} = \frac{nk_BT}{q}\ln(\frac{KI}{I_0}) \quad \text{or} \quad \frac{qV_{oc}}{nk_BT} = \ln(\frac{KI}{I_0})$$

Assuming n = 1, then at two different temperatures T_1 and T_2 but at the same illumination level,

$$\frac{qV_{oc2}}{nk_BT_2} - \frac{qV_{oc1}}{nk_BT_1} = \ln(\frac{KI}{I_{02}}) - \ln(\frac{KI}{I_{01}}) = \ln(\frac{I_{01}}{I_{02}}) \approx \ln(\frac{n_{i1}^2}{n_{i2}^2})$$

We can substitute $n_i^2 = N_c N_v \exp(-E_g / k_BT)$ and neglect the temperature dependences of N_c and N_v compared with the exponential part to obtain

$$\frac{qV_{oc2}}{nk_BT_2} - \frac{qV_{oc1}}{nk_BT_1} = \frac{E_g}{k_B}(\frac{1}{T_2} - \frac{1}{T_1})$$

$$V_{oc2} = V_{oc1}(\frac{T_2}{T_1}) + \frac{nT_2E_g}{q}(\frac{1}{T_2} - \frac{1}{T_1}) = (V_{oc1} - \frac{nE_g}{q})(\frac{T_2}{T_1}) + \frac{nE_g}{q}$$

$$V_{oc2} = (V_{oc1} - \frac{nE_g}{q})(\frac{T_2}{T_1}) + \frac{nE_g}{q} \tag{15}$$

Since E_g = 1.1 eV, $V_{oc1} < nE_g/q$. The first term in the above equation must be negative. Therefore The open circuit voltage and hence output voltage and efficiency of a solar cell increases with decreasing temperature.

Ex. A solar cell has V_{oc1} = 0.55V at 20°C (T_1 = 293K). Assume n = 1. Find V_{oc2} at 60°C (T_1 = 333K).
Sol:

$$V_{oc2} = (V_{oc1} - \frac{E_g}{q})(\frac{T_2}{T_1}) + \frac{E_g}{q} = (0.55 - 1.1)(\frac{333}{293}) + 1.1 = 0.475V$$

The open circuit voltage decreased 13.6%.

Temperature effect is a disadvantage of crystalline Si solar cells compared with thin film solar cells.

References

1. S.O. Kasap, *Optoelectronics and Photonics: Principles and Practices*, Prentice Hall, 2001.
2. S.M. Sze, *Physics of Semiconductor Devices*, 2nd Edition, John Wiley & Sons (1981).