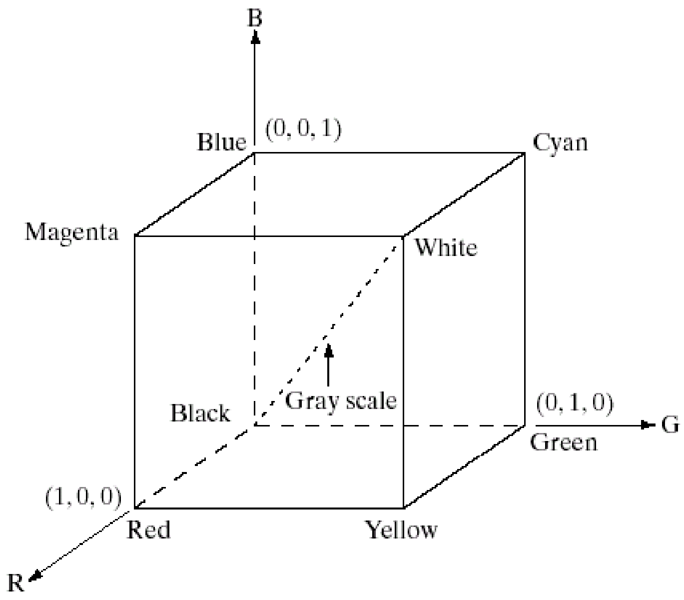
Chap 2

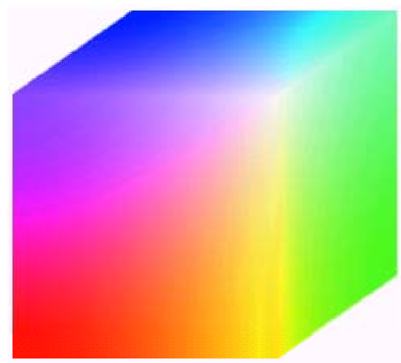
Image Formation

Color Tranformation

- RGB model

FIGURE 6.7 Schematic of the RGB color cube. Points along the main diagonal have gray values, from black at the origin to white at point (1, 1, 1).

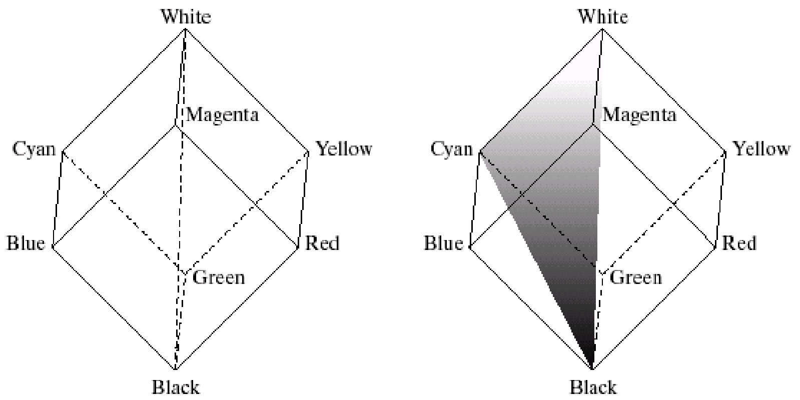




[Gonzalez]

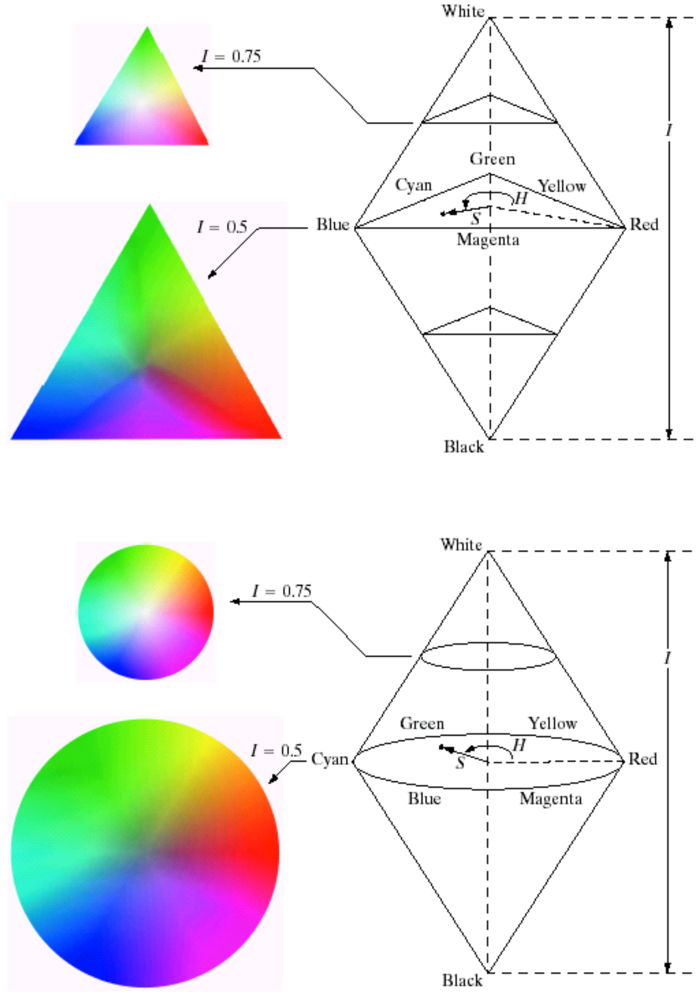
Color Transformation

- HSI model



a b

FIGURE 6.12 Conceptual relationships between the RGB and HSI color models.



a
b

FIGURE 6.14 The HSI color model based on (a) triangular and (b) circular color planes. The triangles and circles are perpendicular to the vertical intensity axis.

[Gonzalez]

Color-transformation

- RGB-HIS (Normalized values, [0~1])

$$H = \begin{cases} \theta & \text{if } B \le G \\ 360 - \theta & \text{if } B > G \end{cases}$$

with

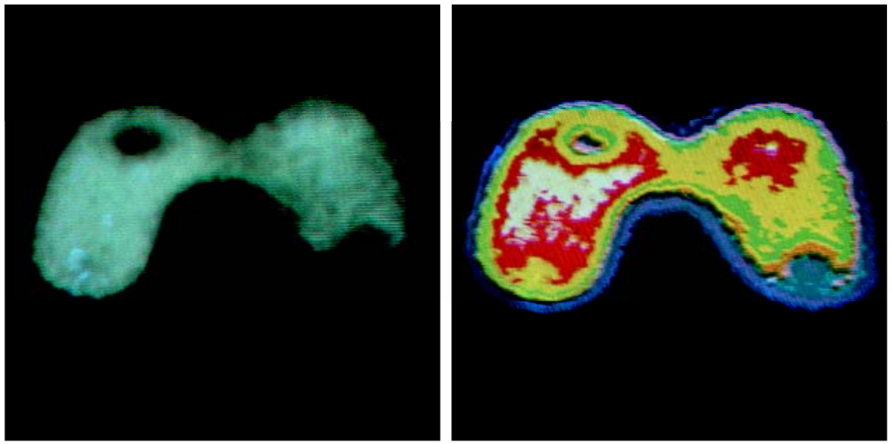
$$\theta = \cos^{-1}\left\{ \frac{\frac{1}{2}\left[(R-G)+(R-B)\right]}{\left[(R-G)^2+(R-B)(G-B)\right]^{1/2}} \right\}$$

$$S = 1 - \frac{1}{(R+G+B)}\left[\min(R,G,B)\right]$$

$$I = \frac{1}{3}(R+G+B)$$

Pseudo-color transformation

- Block-and-white (gray values) image can be converted to be Pseudo-color image
- Intensity slicing



a b

FIGURE 6.20 (a) Monochrome image of the Picker Thyroid Phantom. (b) Result of density slicing into eight colors. (Courtesy of Dr. J. L. Blankenship, Instrumentation and Controls Division, Oak Ridge National Laboratory.)

Image Formation:
Image Coordinate System

An object f(α,β,γ) is mapped into an image g(x,y,z) through a linear image formation system h, a transformation involving translation and rotation, may be established as

G=**R**(**F**-**T**)

where **G** and **F** are respectively, image and object domain coordinate systems, **R** and **T** are, respectively, rotation and translation matrices.

It should be noted that the sequence of rotational operations is important because these operations are not commutative.

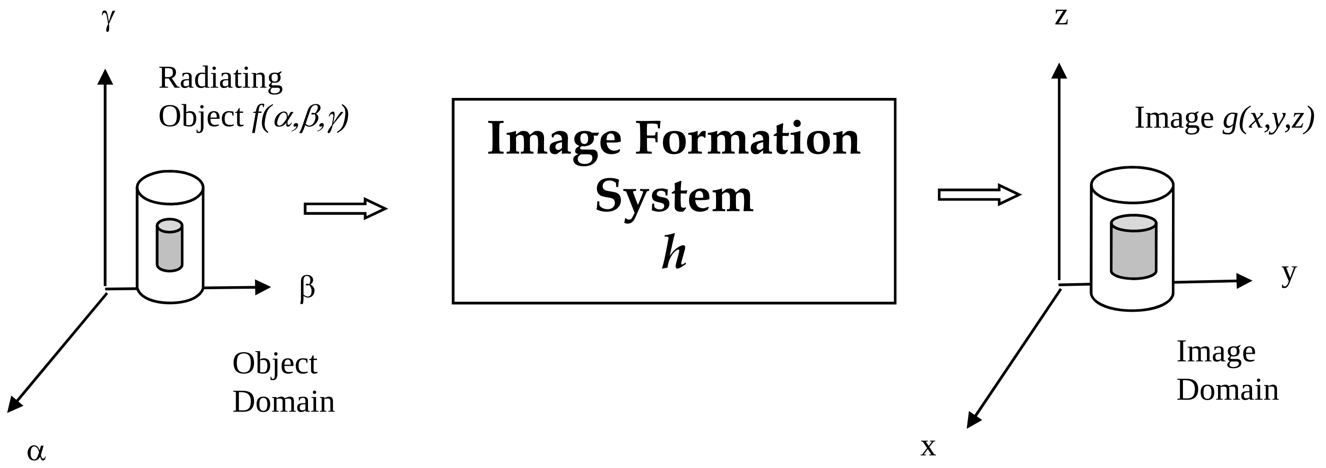


Image Formation:
Image Coordinate System

1. Rotation of f(α,β,γ) about β by an angle θ such that $\mathbf{G}_1(\zeta,\tau,\sigma) = \mathbf{R}_\theta f(\alpha,\beta,\gamma)$

where $R_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ $\quad f = \begin{bmatrix} f_\alpha \\ f_\beta \\ f_\gamma \end{bmatrix}$

物體沿著β軸順時針旋轉θ角

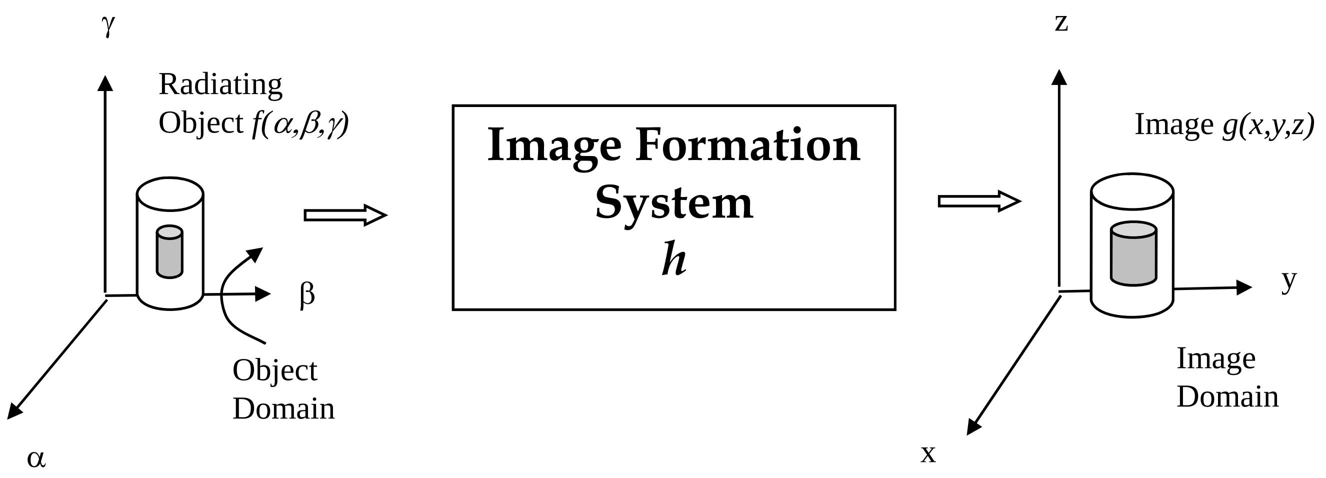


Image Formation:
Image Coordinate System

2. Rotation of $\mathbf{G}_1(\zeta,\tau,\sigma)$ about α by an angle ψ such that $\mathbf{G_2}(\omega,\varepsilon,\nu)= \mathbf{R}_\psi\mathbf{G}_1(\zeta,\tau,\sigma)$

where $$R_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \quad G_{\mathbf{1}} = \begin{bmatrix} g_{1\zeta} \\ g_{1\tau} \\ g_{1\sigma} \end{bmatrix}$$

物體沿著α軸順時針旋轉ψ角

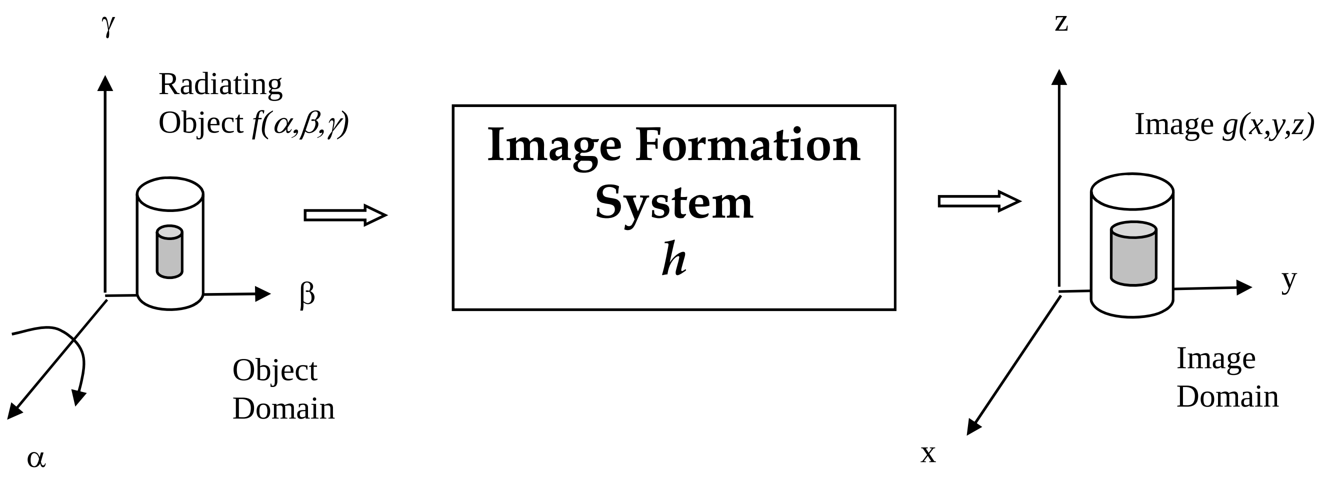


Image Formation:
Image Coordinate System

1. Rotation of $\mathbf{G_2}(\omega,\varepsilon,\nu)$ about γ by an angle ψ such that $\mathbf{G}(x,y,z) = \mathbf{R}_\psi \mathbf{G_2}(\omega,\varepsilon,\nu)$

where $R_\psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad G_2 = \begin{bmatrix} G_{2\omega} \\ G_{2\varepsilon} \\ G_{2\nu} \end{bmatrix}$

物體沿著γ軸順時針旋轉ψ角

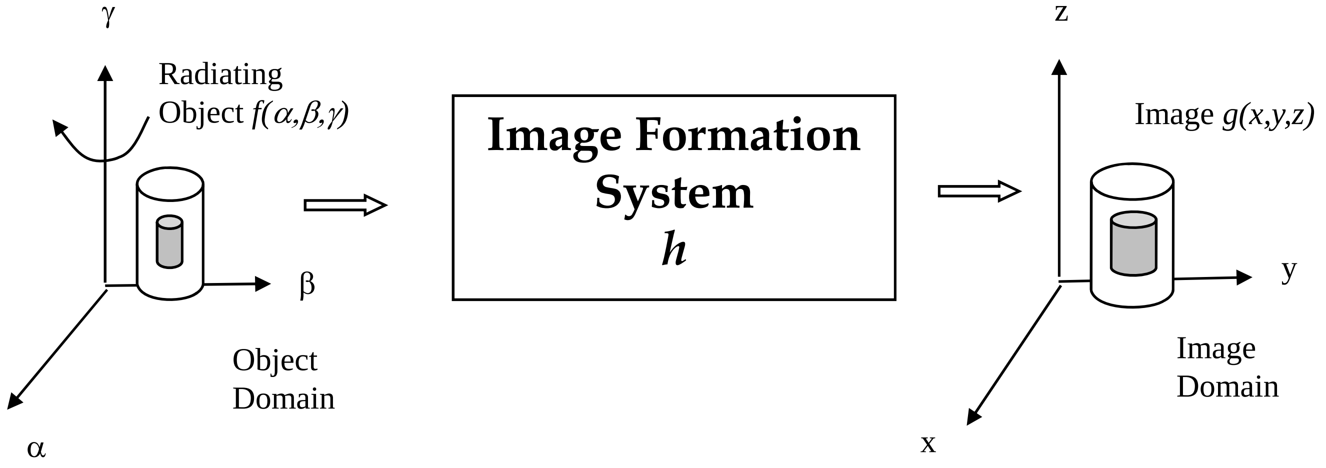


Image Formation:

External Source

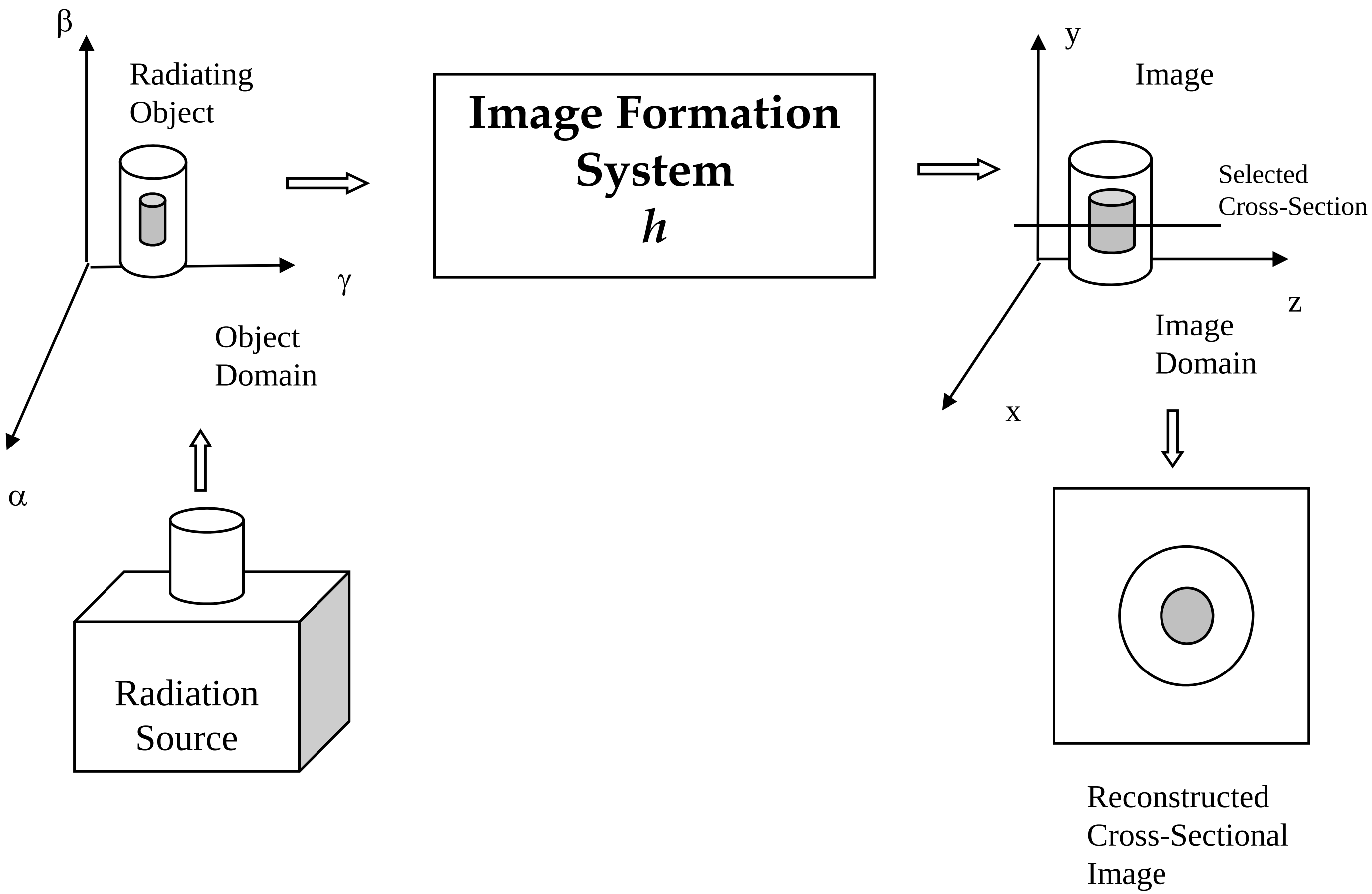
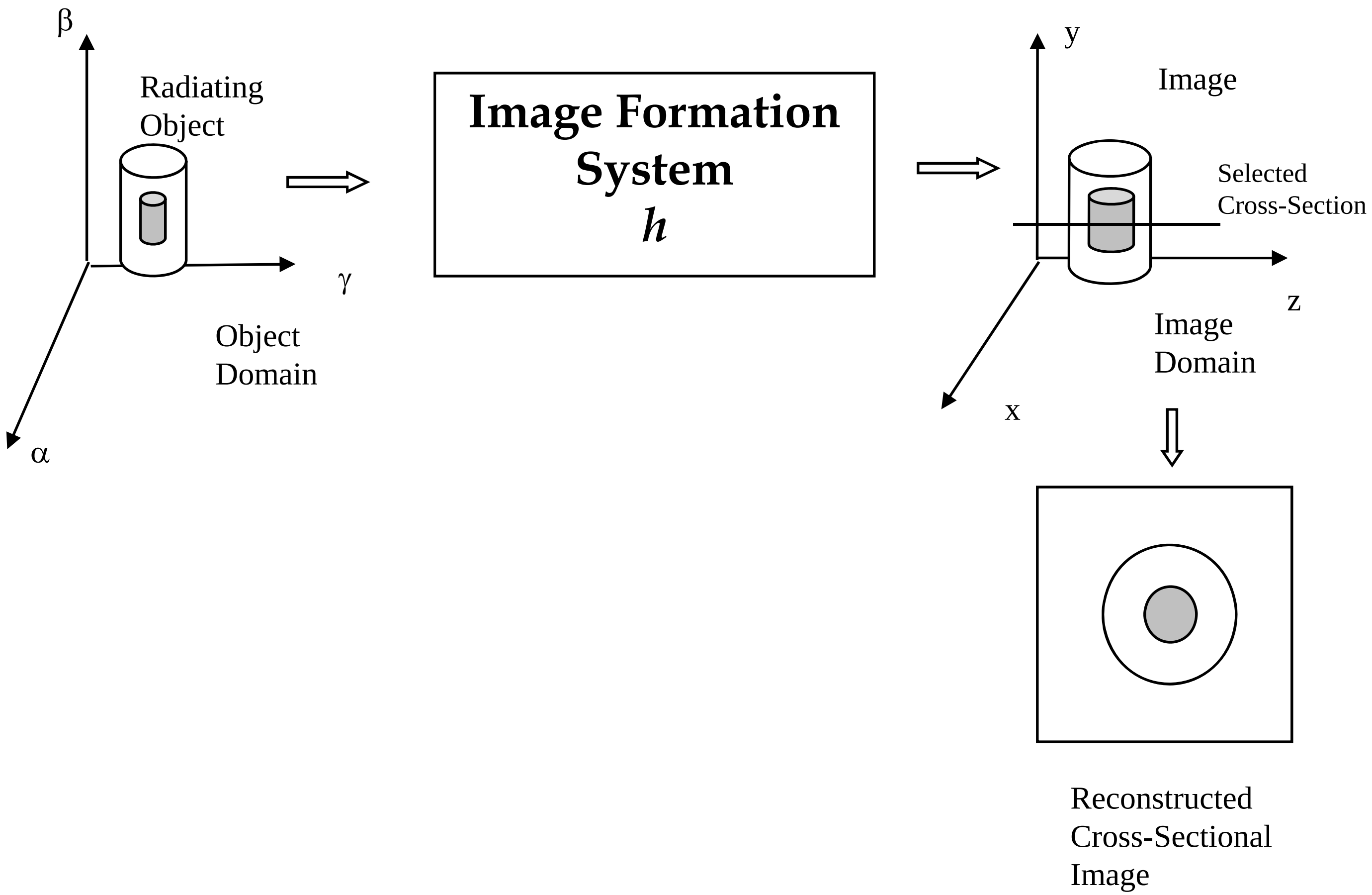


Image Formation:
Internal Source

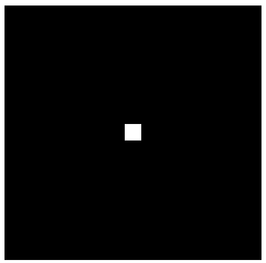


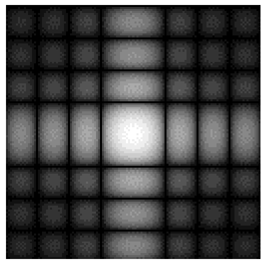
Fourier Transform

1. Provide information about the frequency spectrum
2. Linear transform

$$G(u,v) = FT\{g(x,y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$g(x,y) = FT^{-1}\{G(u,v)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(u,v)e^{j2\pi(ux+vy)}dudv$$





Fourier Transform: Important Properties

Important Properties

- Linearity:

$$FT\{ag(x,y)+bh(x,y)\}=aFT\{g(x,y)\}+bFT\{h(x,y)\}$$

- Scaling: It provides a proportional scaling

$$FT\{g(ax,yb)\}=\frac{1}{ab}G\left(\frac{u}{a},\frac{v}{b}\right)$$

- Translation: Translation in the spatial domain introduces a linear phase shift in the frequency domain

$$FT\{g(x-a,y-b)\}=G(u,v)e^{-j2\pi(ua+vb)}$$

Fourier Transform: Important Properties

4. Convolution: The convolution in spatial domain is represented by multiplication of the respective spectra in the frequency domain

$$FT\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta\right\} = G(u,v)H(u,v)$$

5. Cross-correlation: The operation in the spatial domain is represented by the multiplication of one spectrum with the conjugate of the other spectrum in the frequency domain

$$FT\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\alpha,\beta)h(x+\alpha,y+\beta)d\alpha d\beta\right\} = G(u,v)H^*(u,v)$$

6. Auto-correlation: The Fourier transform of the auto-correlation of a function in the spatial domain is equal to the square of the absolute value of its Fourier transform

$$FT\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\alpha,\beta)g(x+\alpha,y+\beta)d\alpha d\beta\right\} = G^*(u,v)G(u,v) = |G(u,v)|^2$$

Fourier Transform: Important Properties

7. Parseval's Theorem: The total energy is conserved in both spatial and frequency domains

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)g^*(x,y)\,dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(u,v)G^*(u,v)\,dudv$$

8. Separability: If g(x,y) is separable in x and y dimensions, the Fourier transform of g(x,y) will also be separable

$$g(x,y) = g_x(x)g_y(y)$$

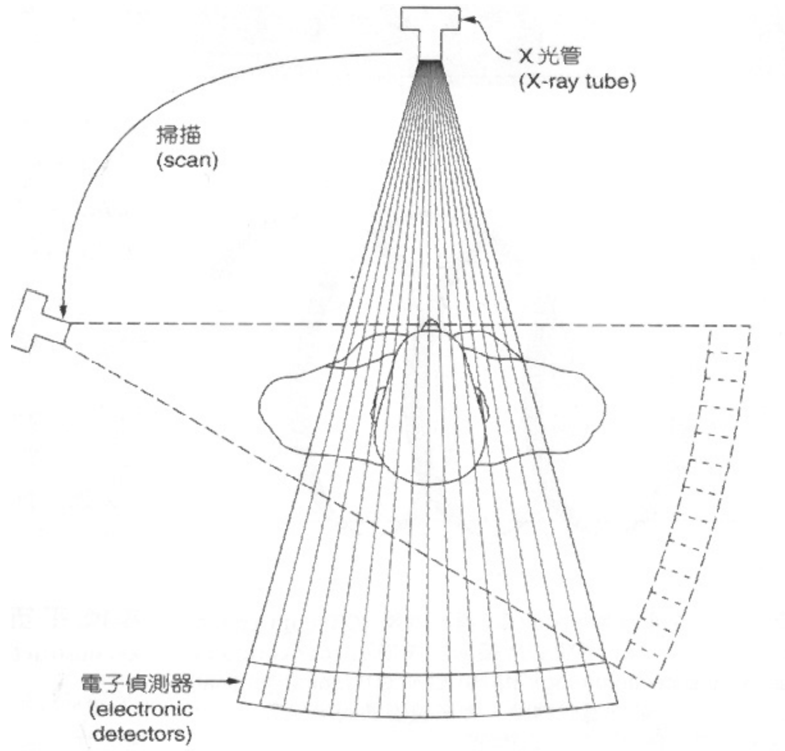
then

$$FT\{g(x,y)\} = FT_x\{g_x(x)\}FT_y\{g_y(y)\}$$

CT (Computed Tomography)

CT的原理

- X光管和偵測器繞著病人一定，產生構成身體不同部份的X光吸收值圖樣



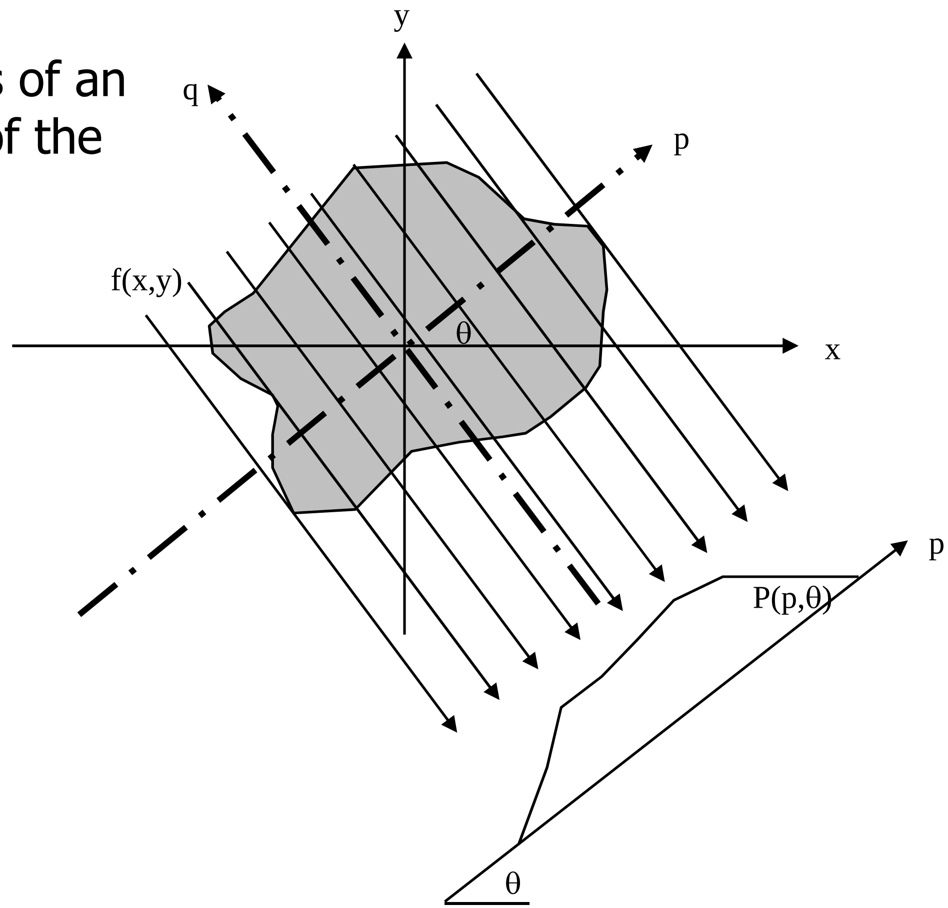
Radon Transform

Radon Transform defines projections of an object mapping the spatial domain of the object to its projection space

$$P(p,\theta) = R\{f(x,y)\} = \int_L f(x,y)dl$$

where the line integral $\int_L$ is defined along the path L

$$x\cos\theta + y\sin\theta = p$$

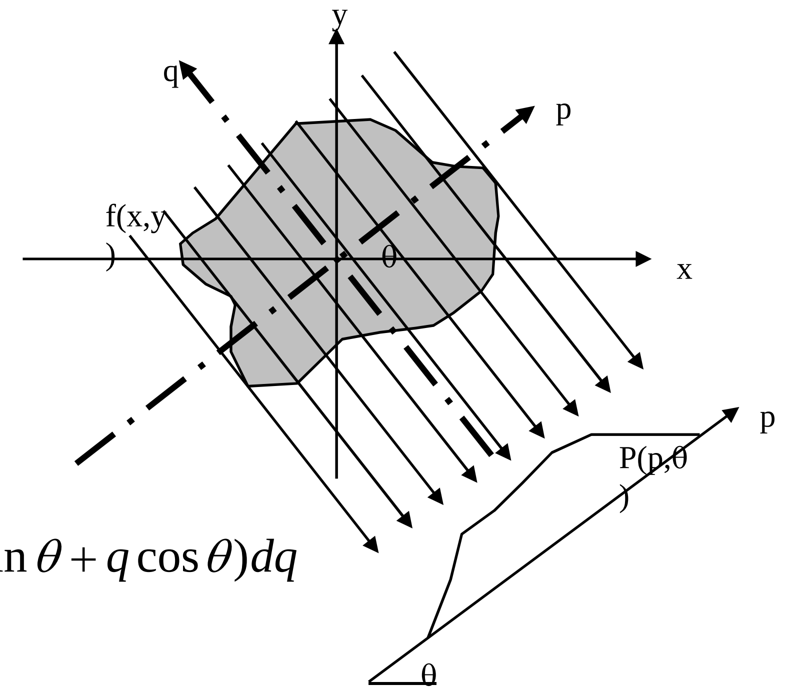


Line integral projection P(p,θ) of the two-dimensional Radon transform.

Radon Transform

The polar coordinate system (p, θ) can be converted into rectangular coordinates in the Radon domain by using a rotated coordinate system(p,q) as

$$x\cos\theta + y\sin\theta = p$$
$$-x\sin\theta + y\cos\theta = q$$

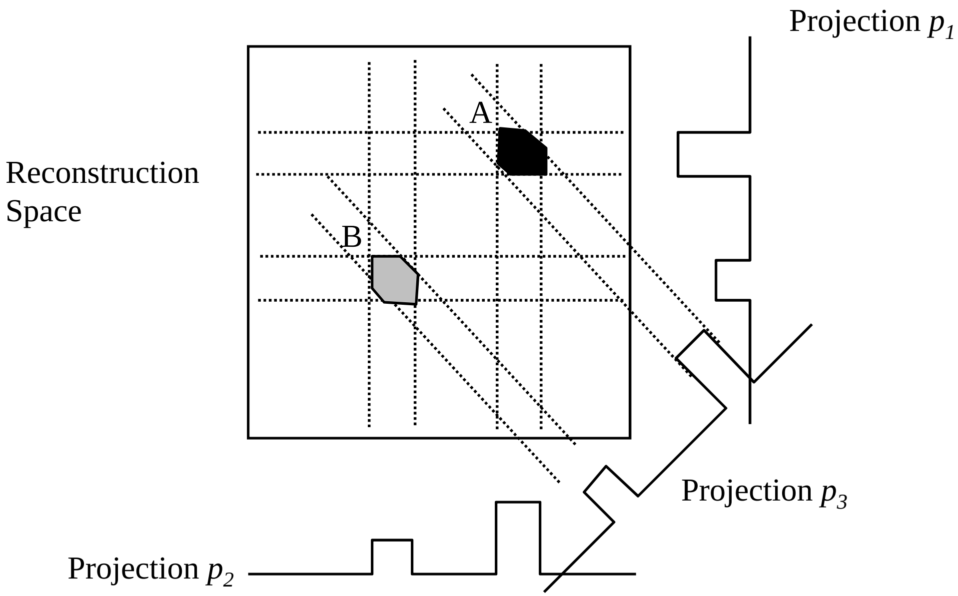


$$R\{f(x,y)\} = J_\theta(p) = \int_{-\infty}^{\infty} f(p\cos\theta - q\sin\theta,\, p\sin\theta + q\cos\theta)\,dq$$

Line integral projection P(p,θ) of the two-dimensional Radon transform.

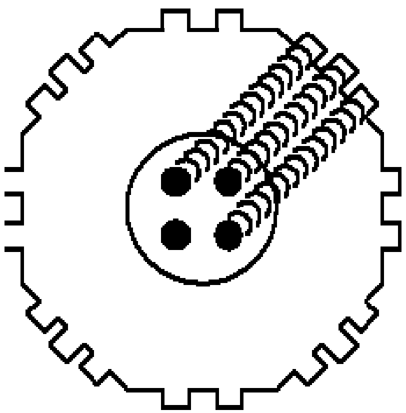
Radon Transform

Back-Projection method



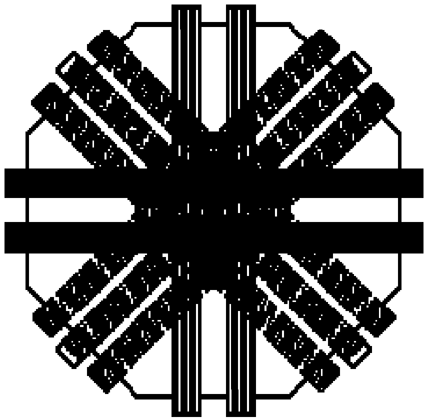
Back Projection

A

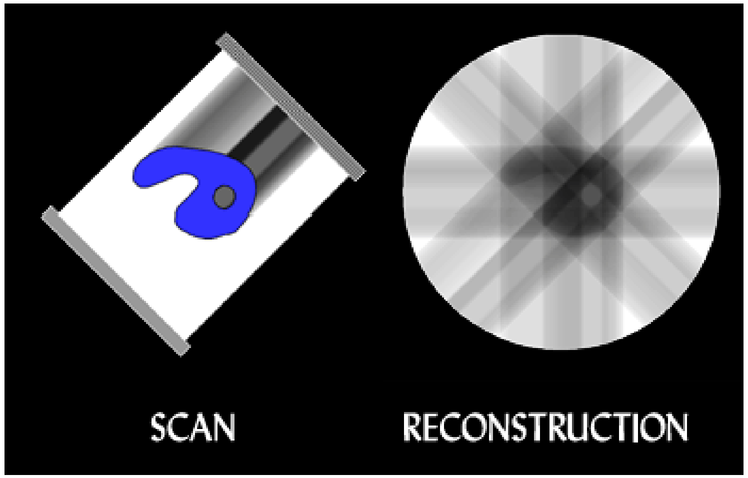


Projection

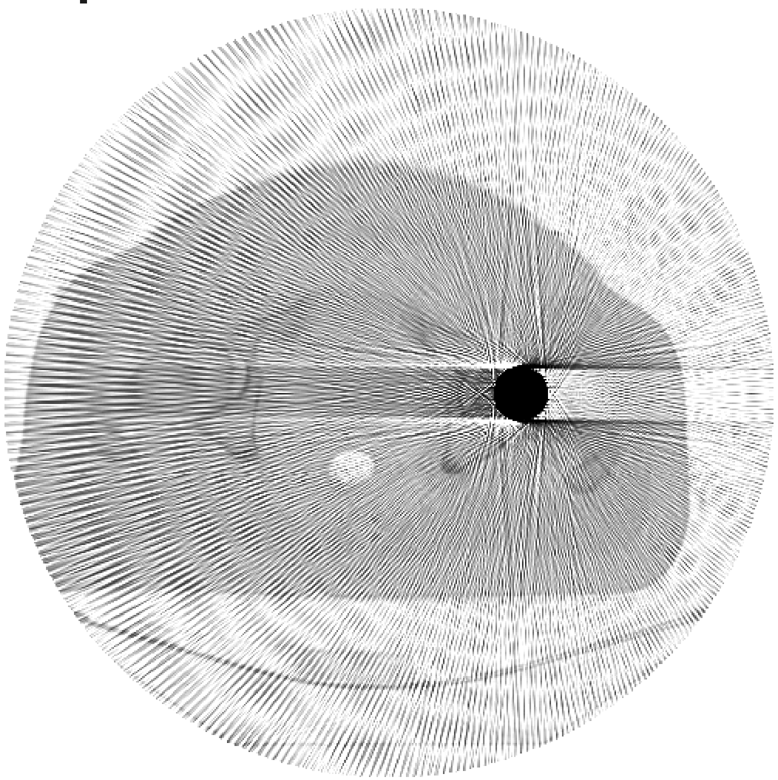
B



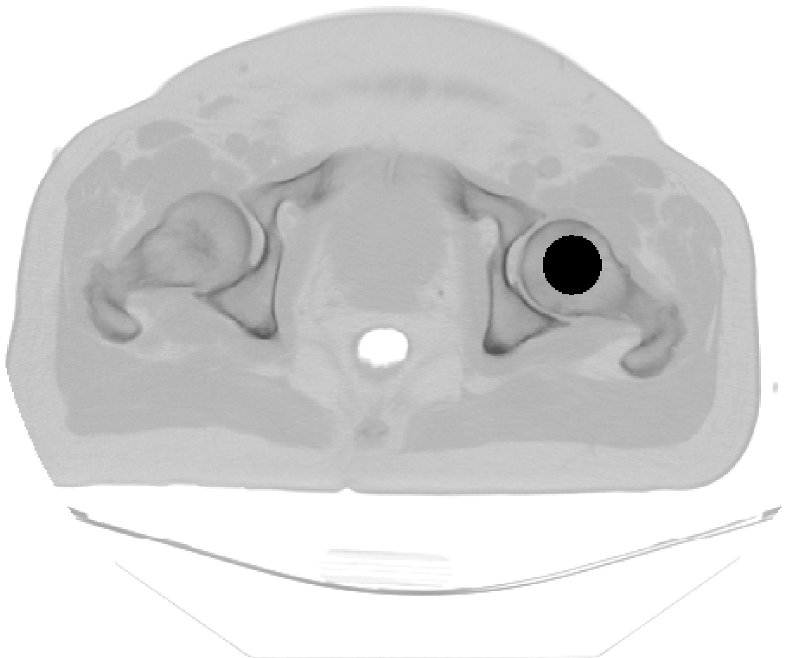
Backprojection



Back Projection



Back Projection



Modified Filtered **Back Projection**