Plane of constant phase

Parallel polarization

Oblique incidence at a dielectric boundary

Incident wave	Reflected wave	Refracted wave

$$\underline{\hat{E}_i} = \hat{E}_{i0} e^{-j\underline{k_i}\cdot\underline{r}}\left(\underline{a_y} \times \underline{a_{ni}}\right) \qquad \underline{\hat{E}_r} = \hat{E}_{r0} e^{-j\underline{k_r}\cdot\underline{r}}\left(\underline{a_y} \times \underline{a_{nr}}\right) \qquad \underline{\hat{E}_t} = \hat{E}_{t0} e^{-j\underline{k_t}\cdot\underline{r}}\left(\underline{a_y} \times \underline{a_{nt}}\right)$$

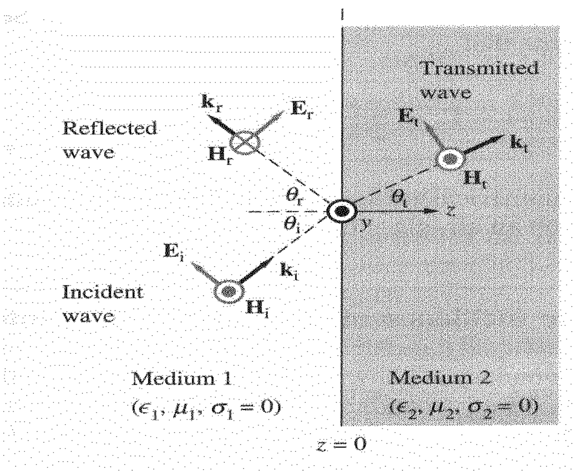
$$\underline{\hat{H}_i} = \frac{\hat{E}_{i0} e^{-j\underline{k_i}\cdot\underline{r}}}{\eta_1}\underline{a_y} \qquad \underline{\hat{H}_r} = \frac{\hat{E}_{r0} e^{-j\underline{k_i}\cdot\underline{r}}}{\eta_1}\underline{a_y} \qquad \underline{\hat{H}_t} = \frac{\hat{E}_{t0} e^{-j\underline{k_t}\cdot\underline{r}}}{\eta_2}\underline{a_y}$$

$$\underline{a_{ni}} = \sin\theta_i \underline{a_x} + \cos\theta_i \underline{a_z} \qquad \underline{a_{nr}} = \sin\theta_r \underline{a_x} - \cos\theta_r \underline{a_z}$$

$$\underline{k_i} = k_i \sin\theta_i \underline{a_x} + k_i \cos\theta_i \underline{a_z} \qquad \underline{k_r} = k_r \sin\theta_r \underline{a_x} - k_r \cos\theta_r \underline{a_z} \qquad \underline{k_t} = k_t \sin\theta_t \underline{a_x} + k_t \cos\theta_t \underline{a_z}$$

$$\begin{cases} \underline{\hat{E}_i} = \hat{E}_{i0}\left(\cos\theta_i \underline{a_x} - \sin\theta_i \underline{a_z}\right) e^{-j(k_i \sin\theta_i x + k_i \cos\theta_i z)} \\ \underline{\hat{E}_r} = \hat{E}_{r0}\left(\cos\theta_r \underline{a_x} + \sin\theta_r \underline{a_z}\right) e^{-j(k_r \sin\theta_r x - k_r \cos\theta_r z)} \\ \underline{\hat{E}_t} = \hat{E}_{t0}\left(\cos\theta_t \underline{a_x} - \sin\theta_t \underline{a_z}\right) e^{-j(k_t \sin\theta_t x + k_t \cos\theta_t z)} \end{cases}$$

$$\begin{cases} \underline{\hat{H}_i} = \dfrac{\hat{E}_{i0}}{\eta_1} e^{-j(k_i \sin\theta_i x + k_i \cos\theta_i z)} \underline{a_y} \\ \underline{\hat{H}_r} = -\dfrac{\hat{E}_{r0}}{\eta_1} e^{-j(k_r \sin\theta_r x - k_r \cos\theta_r z)} \underline{a_y} \\ \underline{\hat{H}_t} = \dfrac{\hat{E}_{t0}}{\eta_2} e^{-j(k_t \sin\theta_t x + k_t \cos\theta_t z)} \underline{a_y} \end{cases}$$



Apply B.C., at z =0,

$$\underline{a_x}\cdot\underline{\hat{E}_i}\big|_{z=0} + \underline{a_x}\cdot\underline{\hat{E}_r}\big|_{z=0} = \underline{a_x}\cdot\underline{\hat{E}_t}\big|_{z=0}$$

$$\underline{a_y}\cdot\underline{\hat{H}_i}\big|_{z=0} + \underline{a_y}\cdot\underline{\hat{H}_r}\big|_{z=0} = \underline{a_y}\cdot\underline{\hat{H}_t}\big|_{z=0}$$

$$\begin{cases} \left(\hat{E}_{i0} + \hat{E}_{r0}\right)\cos\theta_i = \hat{E}_{t0}\cos\theta_t \\ \dfrac{\hat{E}_{i0}}{\eta_1} - \dfrac{\hat{E}_{r0}}{\eta_1} = \dfrac{\hat{E}_{t0}}{\eta_2} \end{cases} \qquad \begin{cases} \dfrac{\hat{E}_{t0}}{\hat{E}_{i0}} = \left(1 + \dfrac{\hat{E}_{r0}}{\hat{E}_{i0}}\right)\dfrac{\cos\theta_i}{\cos\theta_t} \\ \dfrac{1}{\eta_1} - \dfrac{1}{\eta_1}\left(\dfrac{\hat{E}_{r0}}{\hat{E}_{i0}}\right) = \dfrac{1}{\eta_2}\left(\dfrac{\hat{E}_{t0}}{\hat{E}_{i0}}\right) \end{cases}$$

Reflection coefficient $\Gamma_{//} \equiv \dfrac{\hat{E}_{r0}}{\hat{E}_{i0}} = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \quad \Rightarrow \quad \Gamma_{//} = \dfrac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}$

$$\Gamma_{//} = \frac{-\sqrt{\mu_1/\varepsilon_1}\cos\theta_i + \sqrt{\mu_2/\varepsilon_2}\cos\theta_t}{\sqrt{\mu_1/\varepsilon_1}\cos\theta_i + \sqrt{\mu_2/\varepsilon_2}\cos\theta_t} = \frac{\cos\theta_t - \sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i}{\cos\theta_t + \sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i}$$

$$= \frac{\sqrt{1-\sin^2\theta_t} - \sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i}{\sqrt{1-\sin^2\theta_t} + \sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i} = \frac{\sqrt{1-(\varepsilon_{1r}/\varepsilon_{2r})\sin^2\theta_i} - \sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i}{\sqrt{1-(\varepsilon_{1r}/\varepsilon_{2r})\sin^2\theta_i} + \sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i}$$

$$= \frac{-\cos\theta_i + \sqrt{\varepsilon_{1r}/\varepsilon_{2r}}\sqrt{1-(\varepsilon_{1r}/\varepsilon_{2r})\sin^2\theta_i}}{\cos\theta_i + \sqrt{\varepsilon_{1r}/\varepsilon_{2r}}\sqrt{1-(\varepsilon_{1r}/\varepsilon_{2r})\sin^2\theta_i}} = \frac{-\cos\theta_i + \sqrt{(\varepsilon_{1r}/\varepsilon_{2r}) - (\varepsilon_{1r}/\varepsilon_{2r})^2\sin^2\theta_i}}{\cos\theta_i + \sqrt{(\varepsilon_{1r}/\varepsilon_{2r}) - (\varepsilon_{1r}/\varepsilon_{2r})^2\sin^2\theta_i}}$$

$$= \frac{-(\varepsilon_{2r}/\varepsilon_{1r})\cos\theta_i + \sqrt{(\varepsilon_{2r}/\varepsilon_{1r}) - \sin^2\theta_i}}{(\varepsilon_{2r}/\varepsilon_{1r})\cos\theta_i + \sqrt{(\varepsilon_{2r}/\varepsilon_{1r}) - \sin^2\theta_i}} = \frac{-(n_2/n_1)^2\cos\theta_i + \sqrt{(n_2^2/n_1^2) - \sin^2\theta_i}}{(n_2/n_1)^2\cos\theta_i + \sqrt{(n_2^2/n_1^2) - \sin^2\theta_i}}$$

$$\Gamma_{//} = \frac{-\cos\theta_i + \sqrt{(\varepsilon_{1r}/\varepsilon_{2r})}\sqrt{(\varepsilon_{2r}/\varepsilon_{1r}) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\varepsilon_{1r}/\varepsilon_{2r})}\sqrt{(\varepsilon_{2r}/\varepsilon_{1r}) - \sin^2\theta_i}} \Rightarrow \Gamma_{//} = \frac{-(n_2/n_1)^2\cos\theta_i + \sqrt{(n_2^2/n_1^2) - \sin^2\theta_i}}{(n_2/n_1)^2\cos\theta_i + \sqrt{(n_2^2/n_1^2) - \sin^2\theta_i}}$$

Transmission coefficient $\Im_{//} = \frac{\hat{E}_{t0}}{\hat{E}_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} \Rightarrow \Im_{//} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}$

$$\Im_{//} = \frac{2\cos\theta_i}{\cos\theta_t + \sqrt{\varepsilon_2/\varepsilon_1}\cos\theta_i} = \frac{2\cos\theta_i}{\sqrt{1-\sin^2\theta_t} + \sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i}$$

$$= \frac{2\cos\theta_i}{\sqrt{\varepsilon_{2r}/\varepsilon_{1r}}\cos\theta_i + \sqrt{1-(\varepsilon_{1r}/\varepsilon_{2r})\sin^2\theta_i}} = \frac{2\sqrt{(\varepsilon_{1r}/\varepsilon_{2r})}\cos\theta_i}{\cos\theta_i + \sqrt{(\varepsilon_{1r}/\varepsilon_{2r})}\sqrt{1-(\varepsilon_{1r}/\varepsilon_{2r})\sin^2\theta_i}}$$

$$= \frac{2\sqrt{(\varepsilon_{2r}/\varepsilon_{1r})}\cos\theta_i}{(\varepsilon_{2r}/\varepsilon_{1r})\cos\theta_i + \sqrt{(\varepsilon_{2r}/\varepsilon_{1r}) - \sin^2\theta_i}} = \frac{2(n_2/n_1)\cos\theta_i}{(n_2^2/n_1^2)\cos\theta_i + \sqrt{(n_2^2/n_1^2) - \sin^2\theta_i}}$$

$$\Im_{//} = \frac{2\sqrt{(\varepsilon_{1r}/\varepsilon_{2r})}\cos\theta_i}{\cos\theta_i + \sqrt{(\varepsilon_{1r}/\varepsilon_{2r})}\sqrt{1-(\varepsilon_{1r}/\varepsilon_{2r})\sin^2\theta_i}} \Rightarrow \Im_{//} = \frac{2(n_2/n_1)\cos\theta_i}{(n_2^2/n_1^2)\cos\theta_i + \sqrt{(n_2^2/n_1^2) - \sin^2\theta_i}}$$

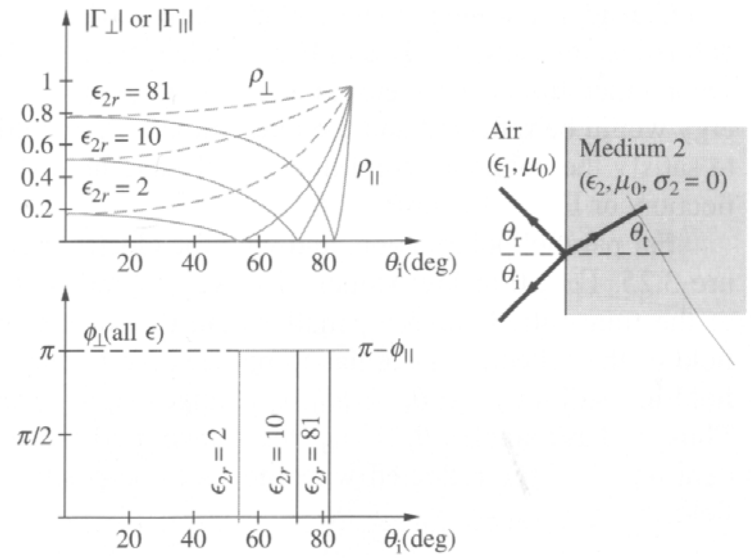


FIGURE 3.30. Reflection coefficient versus the angle of incidence. (a) Magnitude and phase of reflection coefficient for perpendicular ($\Gamma_\perp$) and parallel ($\Gamma_\parallel$) polarization versus angle of incidence θ_i for distilled water ($\epsilon_{2r} = 81$), flint glass ($\epsilon_{2r} = 10$) and paraffin ($\epsilon_{2r} = 2$), all assumed to be lossless. For clarity, the complement of the phase angle $\phi_\parallel$ (i.e., $\pi - \phi_\parallel$) is sketched, rather than $\phi_\parallel$ itself. In all cases, $\phi_\parallel = \pi$ for $\theta_i < \theta_{iB}$, and $\phi_\parallel = 0$ for $\theta_i > \theta_{iB}$, while $\phi_\perp = \pi$ for all θ_i.

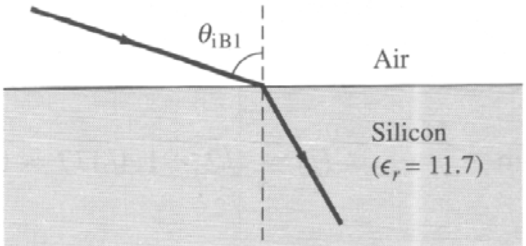
$$1+\Gamma_{//} = \Im_{//}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

Brewster's angle θ_{iB}

If $r_{//} = 0$ (total transmission), then $n_1\cos\theta_t - n_2\cos\theta_i = 0$

$$\Rightarrow \quad \cos\theta_t = (n_2/n_1)\cos\theta_i$$

By Snell's law of refraction, $n_1\sin\theta_i = n_2\sin\theta_t$



$$\Rightarrow \quad \sin\theta_t = (n_1/n_2)\sin\theta_i$$

$$\therefore \quad \sin^2\theta_t + \cos^2\theta_t$$

$$= \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i + \left(\frac{n_2}{n_1}\right)^2 \cos^2\theta_i$$

$$= 1 = \sin^2\theta_i + \cos^2\theta_i$$

$$\left[1-\left(\frac{n_1}{n_2}\right)^2\right]\sin^2\theta_i = \left[\left(\frac{n_2}{n_1}\right)^2 - 1\right]\cos^2\theta_i \quad \left(\text{or } \left(1-\frac{\varepsilon_1}{\varepsilon_2}\right)\sin^2\theta_i = \left(\frac{\varepsilon_2}{\varepsilon_1}-1\right)\cos^2\theta_i\right)$$

$$\tan^2\theta_i = \left(\frac{n_2}{n_1}\right)^2 \quad \Rightarrow \quad \theta_{iB} = \tan^{-1}\frac{n_2}{n_1}$$

for an air-silicon ($\varepsilon_r = 11.7$) interface

$$\theta_{iB} = \tan^{-1}\sqrt{\varepsilon_r} = \tan^{-1}\sqrt{11.7} \approx 73.7^0$$

Normal incidence on a dielectric boundary

$$\theta_i = \theta_r = \theta_t = 0^0$$

$$\begin{cases} \underline{\hat{E}_i} = \hat{E}_{i0} e^{-jk_i z} \underline{a_x} \\ \underline{\hat{E}_r} = \underline{\hat{E}_{r0}} e^{-jk_i z} \underline{a_x} \\ \underline{\hat{E}_t} = \underline{\hat{E}_{t0}} e^{-jk_i z} \underline{a_x} \end{cases} \qquad \begin{cases} \underline{\hat{H}_i} = \dfrac{\hat{E}_{i0}}{\eta_1} e^{-jk_i z} \underline{a_y} \\ \underline{\hat{H}_r} = -\dfrac{\hat{E}_{r0}}{\eta_1} e^{-jk_i z} \underline{a_y} \\ \underline{\hat{H}_t} = \dfrac{\hat{E}_{t0}}{\eta_2} e^{-jk_i z} \underline{a_y} \end{cases}$$

Reflection coefficient $\Gamma_{11} \equiv \dfrac{\hat{E}_{r0}}{\hat{E}_{i0}} = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma_\perp$

Transmission coefficient $\Im_{11} = \dfrac{\hat{E}_{t0}}{\hat{E}_{i0}} = \dfrac{2\eta_2}{\eta_2 + \eta_1} = \Im_\perp$

In most cases, $\mu_1 = \mu_2 = \mu_0$

$$\Gamma = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = \frac{n_1 - n_2}{n_1 + n_2} \qquad \Im = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = \frac{2n_1}{n_1 + n_2}$$

$$\underline{\hat{E}_1} = \underline{\hat{E}_i} + \underline{\hat{E}_r} = \hat{E}_{i0} e^{-jk_i z} \underline{a_x} + \hat{E}_{r0} e^{jk_i z} \underline{a_x} = \hat{E}_{i0} e^{-jk_i z} \underline{a_x} + \Gamma \hat{E}_{i0} e^{jk_i z} \underline{a_x}$$

$$= \hat{E}_{i0} e^{-jk_i z}\left(1 + \Gamma e^{j2k_i z}\right)\underline{a_x}$$

$$\underline{\hat{H}_1} = \underline{\hat{H}_i} + \underline{\hat{H}_r} = \frac{\hat{E}_{i0}}{\eta_1} e^{-jk_i z} \underline{a_y} - \frac{\hat{E}_{r0}}{\eta_1} e^{jk_i z} \underline{a_y} = \frac{\hat{E}_{i0}}{\eta_1} e^{-jk_i z} \underline{a_y} - \Gamma \frac{\hat{E}_{i0}}{\eta_1} e^{jk_i z} \underline{a_y}$$

$$= \frac{\hat{E}_{i0}}{\eta_1} e^{-jk_i z}\left(1 - \Gamma e^{j2k_i z}\right)\underline{a_y}$$

$$\left|\hat{E}_1\right| = \left|\hat{E}_{i0}\right|\left|1 + \Gamma e^{j2k_i z}\right|$$

for $\Gamma > 0$

$\left|\hat{E}_1\right|_{\max} = \left|\hat{E}_{i0}\right|(1 + \Gamma)$ maximum occurs when

$2k_1 z = -2n\pi \qquad n = 0,1,2,\cdots$

$k_1 = 2\pi / \lambda_1 \qquad z = -n\lambda_1 / 2$

$\left|\hat{E}_1\right|_{\min} = \left|\hat{E}_{i0}\right|(1 - \Gamma)$ minima occurs when $2k_1 z = -m\pi \qquad m = 1,3,5\cdots$

$$z = -m\lambda_1 / 4$$

for $\Gamma < 0$

$$\left|\hat{E}_1\right|_{\max} = \left|\hat{E}_{i0}\right|(1-\Gamma)$$

$$\left|\hat{E}_1\right|_{\min} = \left|\hat{E}_{i0}\right|(1+\Gamma)$$

$$\therefore \begin{cases} \left|\hat{E}_1\right|_{\max} = \left|\hat{E}_{i0}\right|(1+|\Gamma|) \\ \left|\hat{E}_1\right|_{\min} = \left|\hat{E}_{i0}\right|(1-|\Gamma|) \end{cases}$$

$$S = \frac{\left|\hat{E}_1\right|_{\max}}{\left|\hat{E}_1\right|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad |\Gamma| = \frac{S-1}{S+1}$$

Oblique incidence on a perfect conductor

Incident wave	Reflected wave

$$\underline{\hat{E}_i} = \underline{\hat{E}_{i0}} e^{-j\underline{k_i}\cdot\underline{r}}\left(\underline{a_y} \times \underline{a_{ni}}\right) \qquad \underline{\hat{E}_r} = \underline{\hat{E}_{r0}} e^{-j\underline{k_r}\cdot\underline{r}}\left(\underline{a_y} \times \underline{a_{nr}}\right)$$

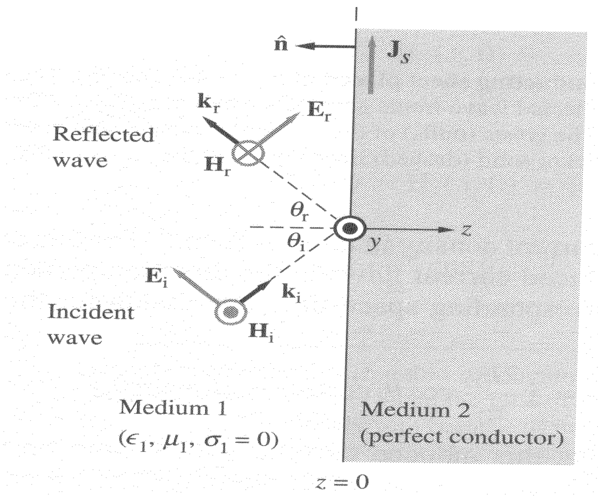
$$\underline{\hat{H}_i} = \frac{\hat{E}_{i0} e^{-j\underline{k_i}\cdot\underline{r}}}{\eta_1}\underline{a_y} \qquad \underline{\hat{H}_r} = \frac{\hat{E}_{r0} e^{-j\underline{k_i}\cdot\underline{r}}}{\eta_1}\underline{a_y}$$

$$\underline{a_{ni}} = \sin\theta_i \underline{a_x} + \cos\theta_i \underline{a_z} \qquad \underline{a_{nr}} = \sin\theta_r \underline{a_x} - \cos\theta_r \underline{a_z}$$

$$\underline{k_i} = k_i \sin\theta_i \underline{a_x} + k_i \cos\theta_i \underline{a_z} \qquad \underline{k_r} = k_r \sin\theta_r \underline{a_x} - k_r \cos\theta_r \underline{a_z}$$

$$\begin{cases} \underline{\hat{E}_i} = \hat{E}_{i0}\left(\cos\theta_i \underline{a_x} - \sin\theta_i \underline{a_z}\right) e^{-j(k_i \sin\theta_i x + k_i \cos\theta_i z)} \\ \underline{\hat{E}_r} = \hat{E}_{r0}\left(\cos\theta_r \underline{a_x} + \sin\theta_r \underline{a_z}\right) e^{-j(k_r \sin\theta_r x - k_r \cos\theta_r z)} \end{cases}$$

$$\begin{cases} \underline{\hat{H}_i} = \frac{\hat{E}_{i0}}{\eta_1} e^{-j(k_i \sin\theta_i x + k_i \cos\theta_i z)} \underline{a_y} \\ \underline{\hat{H}_r} = -\frac{\hat{E}_{r0}}{\eta_1} e^{-j(k_r \sin\theta_r x - k_r \cos\theta_r z)} \underline{a_y} \end{cases}$$



Apply B.C., at z =0,

$$\underline{a_x} \cdot \underline{\hat{E}_i}\big|_{z=0} + \underline{a_x} \cdot \underline{\hat{E}_r}\big|_{z=0} = 0$$

$$\underline{a_y} \cdot \underline{\hat{H}_i}\big|_{z=0} + \underline{a_y} \cdot \underline{\hat{H}_r}\big|_{z=0} = J_s$$

$$\hat{E}_{i0} = -\hat{E}_{r0}, \qquad \theta_i = \theta_r$$

$$\underline{\hat{E}_1} = \underline{\hat{E}_i} + \underline{\hat{E}_r} = \hat{E}_{i0}\cos\theta_i\left(e^{-jk_i\cos\theta_i z} - e^{+jk_i\cos\theta_i z}\right)e^{-jk_i\sin\theta_i x}\,\underline{a_x} - \hat{E}_{i0}\sin\theta_i\left(e^{-jk_i\cos\theta_i z} + e^{+jk_i\cos\theta_i z}\right)e^{-jk_i\sin\theta_i x}\,\underline{a_z}$$

$$= -2\hat{E}_{i0}\left[\cos\theta_i\sin(k_1 z\cos\theta_i)\underline{a_x} - \sin\theta_i\cos(k_1 z\cos\theta_i)\underline{a_z}\right]e^{-jk_i\sin\theta_i x}$$

$$\underline{\hat{H}_1} = \underline{\hat{H}_i} + \underline{\hat{H}_r} = \frac{2\hat{E}_{i0}}{\eta_1}\cos(k_1 z\cos\theta_i z)e^{-jk_i\sin\theta_i x}\,\underline{a_y}$$

Time-average Poynting vector

$$\underline{S_{av}} = \frac{1}{2}\mathrm{Re}\left\{\underline{\hat{E}_1} \times \underline{\hat{H}_1}^*\right\}$$

$$= \frac{2\hat{E}_{i0}^2}{\eta_1}\sin\theta_i\cos^2(k_1\cos\theta_i z)\underline{a_x}$$

at $z = 0$, induced current in the conductor

$$\underline{J_s} = \underline{a_n} \times \underline{H_1}\big|_{z=0} = \frac{2\hat{E}_{i0}}{\eta_1}e^{-jk_i\sin\theta_i x}\,\underline{a_y}$$

Normal incidence on a perfect conductor

$$\theta_i = \theta_t = 0$$

$$\underline{\hat{E}_1} = j2\hat{E}_{i0}\sin(k_1 z)\underline{a_x}$$

$$\underline{\hat{H}_1} = -\frac{2\hat{E}_{i0}}{\eta_1}\cos(k_1 z)\underline{a_y}$$

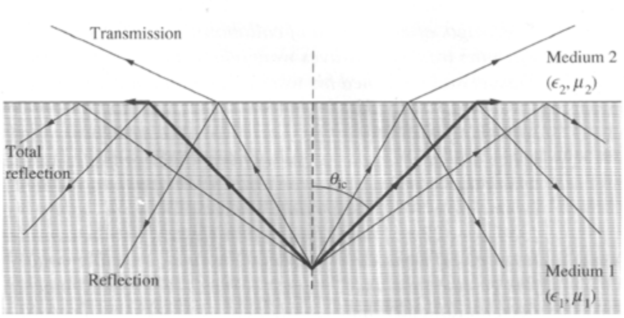
$$\underline{S_{av}} = \frac{1}{2}\mathrm{Re}\left\{\underline{\hat{E}_1} \times \underline{\hat{H}_1}^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\left[j2\hat{E}_{i0}\sin(k_1 z)\right]\underline{a_x} \times \left[-\frac{2\hat{E}_{i0}^*}{\eta_1}\cos(k_1 z)\right]\underline{a_y}\right\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{\left[-j\frac{2\left|\hat{E}_{i0}\right|^2}{\eta_1}\sin(2k_1 z)\right]\underline{a_z}\right\} = 0$$

at $z = 0$, induced current in the conductor

$$\underline{J_s} = \underline{a_n} \times \underline{H_1}\big|_{z=0} = \left[\left(-\underline{a_z}\right)\times\left(\underline{a_y}\right)\right]\frac{2\hat{E}_{i0}}{\eta_1} = \frac{2\hat{E}_{i0}}{\eta_1}\underline{a_x}$$

Total internal reflection



No refraction $\Rightarrow\ \theta_t = 90^o$

$$\Rightarrow \quad n_1 \sin\theta_{ic} = n_2$$

$$\Rightarrow \quad \sin\theta_{ic} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$$

$$\Rightarrow \quad \text{critical angle} \quad \theta_{ic} = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sin^{-1}\frac{n_2}{n_1}$$

if $\theta_i > \theta_{ic} \Rightarrow$ total internal reflection

$$\Rightarrow \quad \frac{n_1}{n_2}\sin\theta_i > 1 \qquad \left(\because \frac{n_1}{n_2}\sin\theta_{ic} = 1\right)$$

$$\Rightarrow \quad \frac{n_1}{n_2}\sin\theta_i = \sin\theta_t > 1 \quad \Rightarrow \quad \theta_t \text{ imaginary (if } \theta_t \text{ is real} \Rightarrow \sin\theta_t \leqq 1\text{)}$$

$$\Rightarrow \quad \cos\theta_t = \sqrt{1-\sin^2\theta_t} = \pm j\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_i - 1} \qquad \text{purely imaginary}$$

$$\Rightarrow \quad \cos\theta_t = -j\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_i - 1} = -j\sqrt{\frac{\varepsilon_{1r}}{\varepsilon_{2r}}\sin^2\theta_i - 1}$$ ensuring the refracted EM field in medium 2 attenuates with increasing distance from the boundary.

(IEEE convention, $e^{j(\omega t - kz)} \Rightarrow e^{-jkz}$) $\Rightarrow (-j)\cdot(-j) = -1 \Rightarrow$

$$\Gamma_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{\cos\theta_i + j\sqrt{\frac{\varepsilon_{2r}}{\varepsilon_{1r}}}\sqrt{\frac{\varepsilon_{1r}}{\varepsilon_{2r}}\sin^2\theta_i - 1}}{\cos\theta_i - j\sqrt{\frac{\varepsilon_{2r}}{\varepsilon_{1r}}}\sqrt{\frac{\varepsilon_{1r}}{\varepsilon_{2r}}\sin^2\theta_i - 1}} = \frac{\cos\theta_i + j\sqrt{\sin^2\theta_i - \frac{\varepsilon_{2r}}{\varepsilon_{1r}}}}{\cos\theta_i - j\sqrt{\sin^2\theta_i - \frac{\varepsilon_{2r}}{\varepsilon_{1r}}}}$$

$$= \frac{\sqrt{\varepsilon_{1r}}\cos\theta_i + j\sqrt{\varepsilon_{1r}\sin^2\theta_i - \varepsilon_{2r}}}{\sqrt{\varepsilon_{1r}}\cos\theta_i - j\sqrt{\varepsilon_{1r}\sin^2\theta_i - \varepsilon_{2r}}} = \frac{n_1\cos\theta_i + j\sqrt{n_1^2\sin^2\theta_i - n_2^2}}{n_1\cos\theta_i - j\sqrt{n_1^2\sin^2\theta_i - n_2^2}}$$

(Optics convention, $e^{-j(\omega t - kz)} \Rightarrow e^{jkz}$) $\Rightarrow j\cdot j = -1$)

$$\Gamma_\perp = \frac{\cos\theta_i - i\sqrt{\sin^2\theta_i - \frac{\varepsilon_{2r}}{\varepsilon_{1r}}}}{\cos\theta_i + i\sqrt{\sin^2\theta_i - \frac{\varepsilon_{2r}}{\varepsilon_{1r}}}} = \frac{\cos\theta_i - i\sqrt{\sin^2\theta_i - \frac{n_2^2}{n_1^2}}}{\cos\theta_i + i\sqrt{\sin^2\theta_i - \frac{n_2^2}{n_1^2}}} = \frac{n_1^2\cos\theta_i - i\sqrt{n_1^2\sin^2\theta_i - n_2^2}}{n_1^2\cos\theta_i + i\sqrt{n_1^2\sin^2\theta_i - n_2^2}}$$

$$\Gamma_{\perp} = \frac{\cos\theta_i + j\sqrt{\sin^2\theta_i - \varepsilon_{21}}}{\cos\theta_i - j\sqrt{\sin^2\theta_i - \varepsilon_{21}}} = e^{j\phi_{\perp}} \qquad \varepsilon_{21} = \frac{\varepsilon_{2r}}{\varepsilon_{1r}}$$

$$e^{j\phi} = z / z^* = e^{j2\delta}, \qquad z = e^{j\delta}$$

$$\Rightarrow \quad \tan\frac{\phi_{\perp}}{2} = \frac{\sqrt{\sin^2\theta_i - \varepsilon_{21}}}{\cos\theta_i}$$

(IEEE convention)

$$\Gamma_{//} = \frac{n_t\cos\theta_t - n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} = -\frac{\sqrt{\frac{\varepsilon_{2r}}{\varepsilon_{1r}}}\cos\theta_i + j\sqrt{\frac{\varepsilon_{1r}}{\varepsilon_{2r}}\sin^2\theta_i - 1}}{\sqrt{\frac{\varepsilon_{2r}}{\varepsilon_{1r}}}\cos\theta_i - j\sqrt{\frac{\varepsilon_{1r}}{\varepsilon_{2r}}\sin^2\theta_i - 1}} = -\frac{\frac{\varepsilon_{2r}}{\varepsilon_{1r}}\cos\theta_i + j\sqrt{\sin^2\theta_i - \frac{\varepsilon_{2r}}{\varepsilon_{1r}}}}{\frac{\varepsilon_{2r}}{\varepsilon_{1r}}\cos\theta_i - j\sqrt{\sin^2\theta_i - \frac{\varepsilon_{2r}}{\varepsilon_{1r}}}} = e^{j\phi_{//}}$$

$$= \frac{\varepsilon_{2r}\cos\theta_i + j\sqrt{\varepsilon_{1r}}\sqrt{\varepsilon_{1r}\sin^2\theta_i - \varepsilon_{2r}}}{\varepsilon_{2r}\cos\theta_i - j\sqrt{\varepsilon_{1r}}\sqrt{\varepsilon_{1r}\sin^2\theta_i - \varepsilon_{2r}}} = \frac{n_2^2\cos\theta_i + jn_1\sqrt{n_1^2\sin^2\theta_i - n_2^2}}{n_2^2\cos\theta_i - jn_1\sqrt{n_1^2\sin^2\theta_i - n_2^2}}$$

(Optics convention)

$$\Gamma_{//} = \frac{n_2^2\cos\theta_i - in_1\sqrt{n_1^2\sin^2\theta_i - n_2^2}}{n_2^2\cos\theta_i + in_1\sqrt{n_1^2\sin^2\theta_i - n_2^2}}$$

$$\Gamma_{//} = -\frac{\varepsilon_{21}\cos\theta_i + j\sqrt{\sin^2\theta_i - \varepsilon_{21}}}{\varepsilon_{21}\cos\theta_i - j\sqrt{\sin^2\theta_i - \varepsilon_{21}}} = e^{j\phi_{//}} \qquad \Rightarrow \qquad \tan\frac{\phi_{//}}{2} = \frac{\sqrt{\sin^2\theta_i - \varepsilon_{21}}}{\varepsilon_{21}\cos\theta_i}$$

linearly polarized wave $\rightarrow$ circularly or elliptically polarized wave by using phase change in total reflection $\Rightarrow$ linearly polarized wave incident at an angle of 45° with respect to the plane of incidence $\Rightarrow$ $|E_{//}| = |E_{\perp}|$

$$\Delta\phi = \phi_{\perp} - \phi_{//} \qquad \Rightarrow \qquad \tan\frac{\Delta\phi}{2} = \frac{\tan(\phi_{\perp}/2) - \tan(\phi_{//}/2)}{1 + \tan(\phi_{\perp}/2)\tan(\phi_{//}/2)} = \frac{\cos\theta_i\sqrt{\sin^2\theta_i - \varepsilon_{21}}}{\sin^2\theta_i}$$

$$\hat{Z}_s = \frac{\hat{E}_t}{\hat{J}_s} = \frac{\hat{E}_y}{\hat{H}_z} =$$