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A study of an integrated inventory with controllable lead time

JASON CHAO-HSIEN PAN†* and JIN-SHAN YANG†

The Japanese experience of Just-in-Time (JIT) production has shown that there are advantages and benefits associated with the efforts to reduce inventory lead time and the associated inventory cost. The length of lead time directly affects the customer service level, inventory investment in safety stock, and the competitive abilities of a business. In most of the literature dealing with inventory problems, either a deterministic model or probabilistic model, lead time is viewed as a prescribed constant or a stochastic variable, and is not subject to control. However, in many practical situations, lead time can be reduced by an additional cost. Moreover, the successful implementation of JIT production in today's supply chain enviromnent requires a new spirit of cooperation between the buyer and the vendor (Goyal and Srinivasan 1992). A desirable condition in long time purchase agreements in such a manufacturing environment is the frequent delivry of small quantities of items so as to minimize inventory holding cost for the buyer. The vendor also needs to minimize his or her total inventory costs. An integrated inventory model that allows the two trading parties to form a strategic alliance for profit sharing may prove helpful in breaking down the traditional barriers. This paper presents an integrated inventory model with controllable lead time. The model is shown to provide a lower total cost and shorter lead time compared with those of Banerjee (1986) and Goyal (1988), and is useful for practical inventory problems.

1. Introduction

In recent years, industries have devoted considerable attention to reducing inventory cost. For example, despite the large scope and numerous benefits of Just-In-Time production systems, which aim to eliminate waste by cutting unnecessary inventory and removing delays in operations, it is the resultant inventory cost reduction that has captured the greatest public attention. In today's supply chain management environment, companies are using the JIT production to gain and maintain a competitive advantage. The characteristics of JIT systems are consistent high quality, small lot sizes, frequent delivery, short lead time, and close supplier ties. Hence, the control of lead time length is one of the key factors to the success of JIT production. Traditionally, the lead time of an inventory model is hypothesized as known (Kim and Park 1985, Ravichandran 1995) or with certain probability distribution (Foote *et al.* 1988). Actually, lead time can be reduced by an additional crashing cost, so as to improve customer service level, and reduce inventory in safety stocks; in other words, it is controllable. The crashing of lead time consists mainly of the following components: order preparation, order transit, supplier lead time, and delivery time (Tersine 1982). Recently, companies have found substantial

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† Department of Industrial Management, National Taiwan University of Science and Technology, 43 Keelung Road, Section 4, Taipei, Taiwan 106.
* To whom correspondence should be addressed. e-mail: chpan@im.ntust.edu.tw

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benefits from establishing a long-term sole-supplier relationship with vendors. By offering a supplier with all of the orders of an item for the next three to five years, the customer can insist on guarantees of reliable delivery, high quality, stable or even decreasing prices, and a share in productivity improvements (Martinich 1997). Both purchaser and vendor may benefit from negotiation and the two sides must determine jointly how to divide the savings (Thomas and Griffin 1996).

Liao and Shyu (1991) presented a continuous review model in which order quantity is predetermined and lead time is a unique decision variable. Ben-Daya and Raouf (1994) extended the Liao and Shyu model (1991) by considering both lead time and the order quantity as decision variables. They derived the optimal lead time and optimal order quantity to minimize the sum of the ordering cost, holding cost, and lead time crashing cost. Ouyang *et al.* (1996) proposed a continuous review inventory model with lead time reduction by allowing shortages to consist of backorders and lost sales. Banerjee (1986) presented a joint economic-lot-size model where a vendor produces to order for a purchaser on a lot-for-lot basis under deterministic conditions. Goyal (1988) further generalized the Banerjee model (1986) by relaxing the assumption of the lot-for-lot policy of the vendor. As a result of using the approach suggested in the Goyal (1988) model, significant reduction in inventory cost can be achieved. Several researchers have shown that in integrated models, one partner's gain exceeds the other partner's loss. Thus, the net benefit can be shared by both parties in some equitable fashion (Goyal and Gupta 1989).

A good vendor will work with a purchaser closely to reduce lead time as much as possible down to a point where it is acceptable to the purchaser, and also reasonable for the vendor to maintain a stable production and delivery schedule (Monczka 1998). The question of who will be charged for the lead time crashing cost is an open issue. Two common schools of thought are (1) the purchaser should be responsible since he is the one using the service, and (2) the vendor and purchaser cooperate in an agreed way. Other methods are possible as long as agreed by both parties. The development of this paper is based on (1) and is proven to be useful.

In this paper, an integrated inventory model with controllable lead time is presented. The proposed model is shown to provide a lower total cost and shorter lead time as compared with those of Banerjee (1986) and Goyal (1988). Goyal (1988) extended Banerjee's (1986) model by relaxing the lot-for-lot production assumption. This paper further extends Goyal's (1988) model by relaxing the production assumption. As soon as a purchaser's lot size is produced, the lot is delivered to the purchaser. The integrated inventory model may prove helpful in breaking down the traditional barriers.

2. Notations and assumptions

To develop the proposed model, the following notations are used:

- D average demand per year,
- P production rate,
- Q order quantity of the purchaser,
- A purchaser's ordering cost per order,
- S vendor's set-up cost per set-up,
- L length of lead time,
- C_V unit production cost paid by the vendor,

C_P unit purchase cost paid by the purchaser,
m an integer representing the number of lots in which the items are delivered from the vendor to the purchaser,
r annual inventory holding cost per dollar invested in stocks.

The assumptions made in the paper are as follows.

(1) The product is manufactured with a finite production rate P, and $P > D$.
(2) The demand X during lead time L follows a normal distribution with mean μL and standard deviation $\sigma\sqrt{L}$.
(3) The reorder point (ROP) equals the sum of the expected demand during lead time and the safety stock.
(4) Inventory is continuously reviewed.
(5) The lead time has n components and these are crashed one component at a time starting with the one with the least crashing cost per unit time, and so on.
(6) The extra costs incurred by the vendor will be fully transferred to the purchaser if shortened lead time is requested.

3. Model formulation

Based an the above notations and assumptions, the total expected annual cost for the purchaser is given by:

$$TEC_P = \text{ordering cost} + \text{holding cost} + \text{lead time crashing cost}.$$

Since A is the ordering cost per order, the expected ordering cost per year is given by $(D/Q)A$. From assumption (3), the reorder point $ROP = \mu L + k\sigma\sqrt{L}$, where k is known as the safety factor. If we assume a linear decrease over the cycle, the average on-hand inventory for the purchaser is given by:

$$\bar{I}_P \cong \frac{Q}{2} + ROP - \mu L = \frac{Q}{2} + k\sigma\sqrt{L}.$$

Hence, the expected holding cost per year is $rC_P(Q/2 + k\sigma\sqrt{L})$.

Liao and Shyu (1991) suggested that lead time can be decomposed into n mutually independent components, each of which has a different crashing cost for reduced lead time, and the crashing cost function is described by a piecewise linear function. The ith component has maximum duration b_i and minimum duration a_i, and a crashing cost per unit time c_i. The components of lead time are crashed one at a time starting with the one of least c_i and so on, and let $\sum_{i=1}^{n} a_i \le L \le \sum_{i=1}^{n} b_i$.

Let L_i be the length of lead time component i crashed to its minimum duration, $i = 1, 2, \ldots, n$, then L_i can be expressed as

$$\begin{aligned} L_i &= \sum_{j=1}^{i} a_j + \sum_{j=i+1}^{n} b_j \\ &= \sum_{j=1}^{i} a_j + \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} b_j \end{aligned}$$

$$= \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} (b_j - a_j)$$

$$= L_0 - \sum_{j=1}^{i} (b_j - a_j),$$

where $L_0 \equiv \sum_{j=1}^{n} b_j$.

The lead time crashing cost $R(L)$ for a given $L \in (L_i, L_{i-1}]$ is given by:

$$R(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j). \tag{1}$$

Therefore, the expected lead time crashing cost per year is $(D/Q)R(L)$.

The total expected annual cost for the purchaser is given by:

$$TEC_P(Q, L) = \frac{D}{Q}A + \left(\frac{Q}{2} + k\sigma\sqrt{L}\right) rC_P + \frac{D}{Q}R(L). \tag{2}$$

For the vendor's inventory model, its total expected annual cost can be represented by:

$$TEC_V = \text{set-up cost} + \text{holding cost}.$$

Since S is the vendor's set-up cost per set-up, and the production quantity for the vendor in a lot will be mQ, its expected set-up cost per year is given by $[D/(mQ)]S$.

The integrated inventory model is designed for a vendor's production situations in which once an order is placed, the production begins and a constant number of units are added to inventory each day until the production run has been completed. The vendor produces the item in the quantity of mQ, and the purchaser would receive it in m lots, with which each having a quantity of Q. The inventory pattern for this model is shown in figure 1.

For the vendor, its average inventory can be evaluated as follows:

$$\bar{I}_v = \left\{ \left[mQ\left(\frac{Q}{P} + (m-1)\frac{Q}{D}\right) - \frac{m^2Q^2}{2P} \right] - \left[\frac{Q}{D}(1 + 2 + \cdots + (m-1))Q \right] \right\} \Big/ \left(\frac{mQ}{D}\right)$$

$$= \frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right). \tag{3}$$

Hence, the vendor's expected holding cost per year is

$$rC_V\left(\frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)\right).$$

It follows that the total expected annual cost for the vendor is:

$$TEC_V(Q, m) = \frac{D}{Qm}S + \frac{Q}{2} rC_V\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right). \tag{4}$$

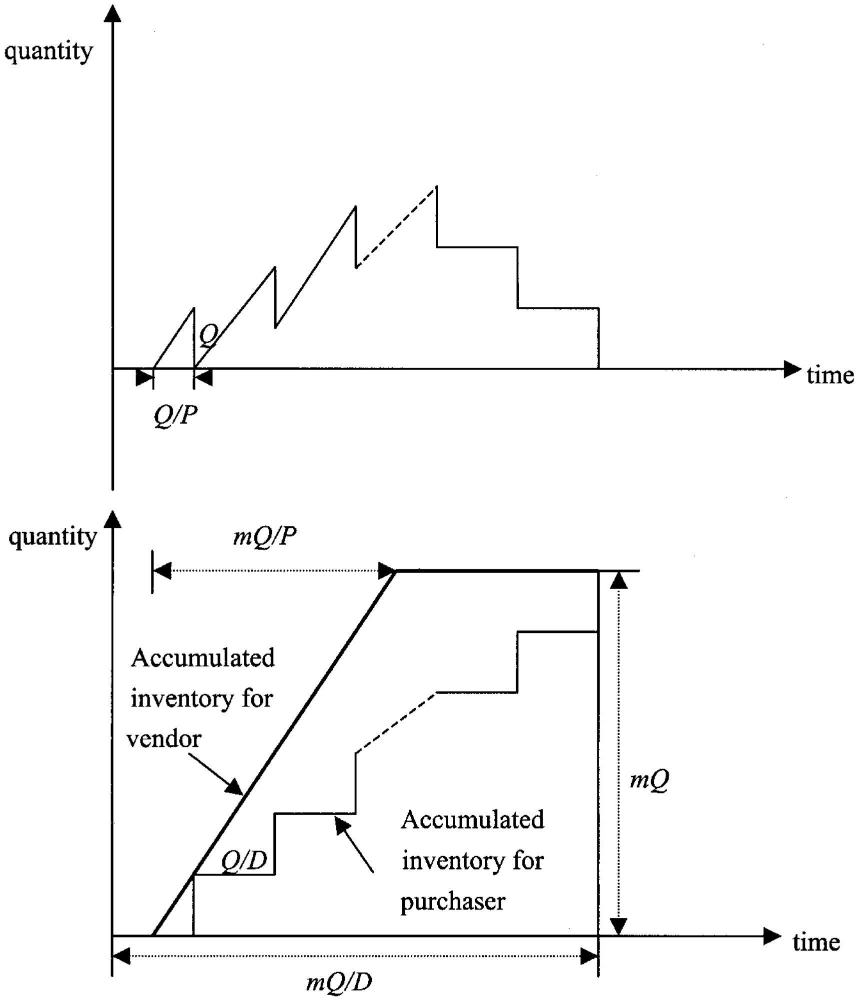

Figure 1. The inventory pattern for the vendor.

If the purchaser's order quantity is Q, and the vendor's lot size is mQ, then the joint total expected annual cost is given by:

$$JTEC(Q, L, m) = \frac{D}{Q}\left[A + \frac{S}{m} + R(L)\right] + \frac{Q}{2}r\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_V + C_P\right] + rC_P k\sigma\sqrt{L}. \tag{5}$$

Taking the partial derivatives of $JTEC(Q, L, m)$ with respect to Q and L in each time interval (L_i, L_{i-1}), and equating them to zero, we obtain:

$$\frac{\partial JTEC(Q, L, m)}{\partial Q} = -\frac{D}{Q^2}\left(A + \frac{S}{m} + R(L)\right) + \frac{r}{2}\left[\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_V + C_P\right] = 0 \tag{6}$$

$$\frac{\partial JTEC(Q,L,m)}{\partial L} = -\frac{D}{Q}c_i + \frac{r}{2}C_P k\sigma L^{-1/2} = 0. \tag{7}$$

Hence, for fixed $L \in (L_i, L_{i-1})$, $JTEC(Q, L, m)$ is convex in Q, since

$$\frac{\partial^2 JTEC(Q,L,m)}{\partial Q^2} = \frac{2D}{Q^3}\left(A + \frac{S}{m} + R(L)\right) > 0.$$

However, for fixed Q, $JTEC(Q, L, m)$ is concave in $L \in (L_i, L_{i-1})$, because

$$\frac{\partial^2 JTEC(Q,L,m)}{\partial L^2} = -\frac{r}{4}C_P k\sigma L^{-3/2} < 0.$$

Therefore, for fixed Q, the minimum joint total expected annual cost will occur at the end points of the interval. From (6), we have

$$Q = \left[\frac{2D\left(A + \frac{S}{m} + R(L)\right)}{r\left(C_V\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + C_P\right)}\right]^{1/2} \qquad L \in (L_i, L_{i-1}). \tag{8}$$

The derivation of equation (8) is shown in Appendix 1.

For a particular value of m, the joint total expected annual cost is described by:

$$JTEC(m) = \left[2Dr\left(A + \frac{S}{m} + R(L)\right)\left(C_V\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + C_P\right)\right]^{1/2} + rC_P k\sigma\sqrt{L}. \tag{9}$$

We can ignore the terms that are independent of m, and take the square of (9). Then, minimizing $JTEC(m)$ is equivalent to minimizing

$$\begin{aligned}(JTEC(m))^2 &= 2Dr\left(A + \frac{S}{m} + R(L)\right)\left(C_V\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + C_P\right)\\ &= 2Dr\left[(A + R(L))\left(C_P - \left(1 - \frac{2D}{P}\right)C_V\right) + SC_V\left(1-\frac{D}{P}\right)\right.\\ &\quad \left. + mC_V(A + R(L))\left(1-\frac{D}{P}\right) + \frac{S}{m}\left(C_P - \left(1-\frac{2D}{P}\right)C_V\right)\right].\end{aligned}$$

Once again, ignoring the terms that are independent of m, the minimization of the problem can be reduced to that of minimizing

$$Z(m) = mC_V(A + R(L))\left(1-\frac{D}{P}\right) + \frac{S}{m}\left(C_P - \left(1-\frac{2D}{P}\right)C_V\right). \tag{10}$$

The optimal value of $m = m^*$ is obtained when

$$Z(m^*) \le Z(m^* - 1) \qquad \text{and} \qquad Z(m^*) \le Z(m^* + 1). \tag{11}$$

On substituting relevant values in (11), we get:

$$m^*(m^*-1) \le \frac{S\left(C_P - \left(1-\frac{2D}{P}\right)C_V\right)}{C_V(A + R(L))\left(1-\frac{D}{P}\right)}$$

and (12)

$$m^*(m^*+1) \geq \frac{S\left(C_P - \left(1 - \frac{2D}{P}\right)C_V\right)}{C_V(A + R(L))\left(1 - \frac{D}{P}\right)}.$$

From (12), the following condition is obtained:

$$m^*(m^*-1) \leq \frac{S\left(C_P - \left(1 - \frac{2D}{P}\right)C_V\right)}{C_V(A + R(L))\left(1 - \frac{D}{P}\right)} \leq m^*(m^*+1). \tag{13}$$

The derivation of equation (13) is shown in Appendix 2.

Thus, we can use the following procedure to find optimal values of m, Q and L.

Step 1. Compute the range of m by equation (13).
Step 2. For each L_i, $i = 1, 2, \ldots, n$, compute Q_i using equation (8).
Step 3. For each (Q_i, L_i, m_i), compute $JTEC(Q_i, L_i, m)$, for $i = 0, 1, \ldots, n$.
Step 4. Set $JTEC(Q^*, L^*, m^*) = \min_{i=0,1,\ldots,n} JTEC(Q_i, L_i, m_i)$. Then (Q^*, L^*, m^*) is an optimal solution.

3.1. *A numerical example*

Consider an inventory system with the following characteristics: $D = 1000$ unit/year, $P = 3200$ unit/year, $A = \$25$/order, $S = \$400$/set-up, $C_P = \$25$/unit, $C_V = \$20$/unit, $r = 0.2$, $k = 2.33$, $\sigma = 7$ unit/week, and the lead time has three components with data shown on table 1 (Banerjee 1986, Goyal 1988, Ouyang *et al.* 1999).

From (13), we have $m = 3$, $m = 4$ and $m = 5$. Applying the above procedure yields optimal integer solutions with $m^* = 4$, optimal lead time $L^* = 42$ days, and optimal order quantity $Q^* = 132$ units. The joint total expected annual cost is \$2114.3. The resulting solution procedure is summarized in table 2.

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

Table 1. Lead time data for the example.

		$m = 3$		$m = 4$		$m = 5$	
I	$R(L_i)$	Q_i	$JTEC(Q_i, L_i, m_i)$	Q_i	$JTEC(Q_i, L_i, m_i)$	Q_i	$JTEC(Q_i, L_i, m_i)$
0	0.0	164	2159.6	131	2134.6	110	2134.0
1	1.4	165	2137.2	132	2114.3*	111	2115.7
2	18.2	173	2200.0	141	2200.9	120	2224.8
3	53.2	190	2370.8	157	2414.5	135	2477.5

* The minimum joint total expected annual cost.

Table 2. Summary of the solution procedure.

	$m = 1$				$m > 1$	
	Purchaser Decision	Vendor Decision	Banerjee's model	This Model#	Goyal's Model ($m = 2$)	This model# ($m = 4$)
Purchaser's order size	100	800	369	369	198	132
Vendor's lot size	100	800	369	369	396	528
Purchaser's annual cost	730.7	2262.0	1221.0	1193.8	852.0	729.7
Vendor's annual cost	4062.5	1000.0	1314.6	1314.6	1653.6	1384.6
Joint total annual cost	4793.2	3262.0	2535.6	2508.4	2505.6	2114.3

With lead time crashing.

Table 3. Summary of the comparison.

Safety stocks are extra inventory kept on hand as a cushion against stock-outs due to random perturbations of nature or the environment (Tersine 1982). Thus, the models should take the safety stock into consideration before computing the solutions. For $m = 1$, the proposed model is shown to provide a lower cost and shorter lead time as compared with those of the model of Banerjee (1986). For m greater than one, the minimum annual cost is \$2505.6 in the Goyal (1988) model, and is only \$2114.3 in this article. The results of the computations by the models are presented in table 3.

Both purchaser and vendor determine inventory policy independently. The purchaser computes his economic order quantity using equation (2). To obtain the minimum cost lot size, take the first partial derivatives of $TEC_P(Q, L)$ with respect to Q and L in each time interval (L_i, L_{i-1}), and set them to zero; thus:

$$\frac{\partial TEC_P(Q, L)}{\partial Q} = -\frac{D}{Q^2}(A + R(L)) + \frac{r}{2}C_P = 0 \tag{14}$$

$$\frac{\partial TEC_P(Q, L)}{\partial L} = -\frac{D}{Q}c_i + \frac{r}{2}C_P k\sigma L^{-1/2} = 0. \tag{15}$$

Hence, for fixed $L \in (L_i, L_{i-1})$, $TEC_P(Q, L)$ is convex in Q, since

$$\frac{\partial^2 TEC_P(Q, L)}{\partial Q^2} = \frac{2D}{Q^3}(A + R(L)) > 0.$$

However, for fixed Q, $TEC_P(Q, L)$ is concave in $L \in (L_i, L_{i-1})$, because

$$\frac{\partial^2 TEC_P(Q, L)}{\partial L^2} = -\frac{r}{4}C_P k\sigma L^{-3/2} < 0.$$

Therefore, for fixed Q, the minimum total expected annual cost for the purchaser will occur at the end points of the interval. From (14), we have

$$Q = \left[\frac{2D(A + R(L))}{rC_P}\right]^{1/2} \qquad L \in (L_i, L_{i-1}). \tag{16}$$

Model type	Purchaser		Vendor	
Independent	Order quantity	103	Production quantity	515
	Total annual cost	\$713.6	Total annual cost	\$1407.5
Integrated	Order quantity	132	Production quantity	528
	Total annual cost	\$729.7	Total annual cost	\$1384.6
	Allocated total annual cost	\$711.3	Allocated total annual cost	\$1403.0

Table 4. Allocation of the total annual cost.

From (16), it is clear that the optimal policy is to order $Q = 103$ units and the optimal lead time is $L = 42$ days at a total annual cost of \$713.6.

The economic production quantity will be an integer multiple of the buyer's purchase quantity. In (4) the only unknown variable is m. Let $m = 1, 2, 3, \ldots,$ and choose the one for which equation (4) is minimized. The vendor yields $m = 5$ since $Q = 103$ with corresponding total annual cost of \$1407.5. Therefore the joint total annual cost is \$2121.1. Goyal (1976) suggested that the total annual cost should be allocated to the vendor and the purchaser as follows:

$$\beta = \frac{TEC_P(Q^*, L^*)}{TEC_P(Q^*, L^*) + TEC_V(Q^*, m^*)}, \tag{17}$$

$$\text{Cost to the purchaser} = \beta[JTEC(Q^*, L^*, m^*)],$$

$$\text{Cost to the vendor} = (1 - \beta)[JTEC(Q^*, L^*, m^*)].$$

The resulting allocation is summarized in table 4. The vendor should compensate the purchaser \$18.4 per year. Much of the literature has dealt with the interaction between a vendor and a purchaser (Goyal 1976, Goyal and Gupta 1989). The integrated model can contribute significantly to better the vendor–purchaser relationship.

4. Conclusions

Lead time is an important element in any inventory management system. By shortening the lead time, we can lower the safety stock, reduce the loss caused by stockout, improve customer service level, and increase the competition ability in business (Ouyang and Wu 1997). In many practical situations, lead time can be reduced by an additional crashing cost. That is, lead time is controllable.

This article presents an integrated inventory model for minimizing the sum of the ordering cost, holding cost and crashing cost. A procedure is provided to find the optimal order quantity, lead time and delivering number when the probability distribution of the lead time demand is normal.

The production assumptions of Banerjee (1986) and Goyal (1988) are restrictive in nature. In this paper, the integrated inventory model with controllable lead time is suggested and it is shown to provide a lower total cost and shorter lead time as compared with the models of Banerjee (1986) and Goyal (1988). By adopting a jointly optimal ordering policy, one partner's gain exceeds the other's loss, and the net benefit can be shared by both parties in some equitable fashion.

Assumption 6 in the model may be contentious and other ways of charging for lead time crashing cost are currently being studied.

Appendix 1: Derivation of equation (8)

From equation (6), it follows that:

$$\frac{D}{Q^2}\left(A+\frac{S}{m}+R(L)\right)=\frac{r}{2}\left[\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)C_V+C_P\right].$$

Solving for Q, we have

$$Q^2=\frac{2D\left(A+\frac{S}{m}+R(L)\right)}{r\left(C_V\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)+C_P\right)}$$

$$Q=\left[\frac{2D\left(A+\frac{S}{m}+R(L)\right)}{r\left(C_V\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)+C_P\right)}\right]^{1/2}.$$

Appendix 2: Derivation of equation (13)

From equation (10), the optimal value of $m = m^*$ is obtained when

$$Z(m^*) \le Z(m^*-1) \qquad \text{and} \qquad Z(m^*) \le Z(m^*+1).$$

Substituting with relevant values, we have, respectively,

$$mC_V(A+R(L))\left(1-\frac{D}{P}\right)+\frac{S}{m}\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right)$$
$$\le (m-1)C_V(A+R(L))\left(1-\frac{D}{P}\right)+\frac{S}{m-1}\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right)$$

and

$$mC_V(A+R(L))\left(1-\frac{D}{P}\right)+\frac{S}{m}\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right)$$
$$\le (m+1)C_V(A+R(L))\left(1-\frac{D}{P}\right)+\frac{S}{m+1}\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right).$$

Then,

$$(m-(m-1))C_V(A+R(L))\left(1-\frac{D}{P}\right)\le\left(\frac{1}{m-1}-\frac{1}{m}\right)S\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right)$$

and

$$((m+1)-m)C_V(A+R(L))\left(1-\frac{D}{P}\right)\ge\left(\frac{1}{m}-\frac{1}{m+1}\right)S\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right).$$

Accordingly, it follows that

$$C_V(A+R(L))\left(1-\frac{D}{P}\right)\le\frac{1}{m(m-1)}S\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right)$$

and

$$C_V(A+R(L))\left(1-\frac{D}{P}\right)\ge\frac{1}{m(m+1)}S\left(C_P-\left(1-\frac{2D}{P}\right)C_V\right).$$

Equivalently,

$$m^*(m^*-1) \le \frac{S\left(C_P - \left(1 - \frac{2D}{P}\right)C_V\right)}{C_V(A+R(L))\left(1-\frac{D}{P}\right)}$$

and

$$m^*(m^*+1) \ge \frac{S\left(C_P - \left(1 - \frac{2D}{P}\right)C_V\right)}{C_V(A+R(L))\left(1-\frac{D}{P}\right)}.$$

Finally, it is concluded that

$$m^*(m^*-1) \le \frac{S\left(C_P - \left(1 - \frac{2D}{P}\right)C_V\right)}{C_V(A+R(L))\left(1-\frac{D}{P}\right)} \le m^*(m^*+1).$$

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