International Journal of Systems Science
Vol. 36, No. 8, 20 June 2005, 485–490

An infinite ε-bound stability criterion for a class of multiparameter singularly perturbed time-delay systems

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(*Received 27 November 2003; revised 9 July 2004; accepted 28 April 2005*)

This paper presents an adjustable singular perturbed parameter approach, which is based on the Lyapunov stability theorem, in the stability analysis of multi-parameter singularly perturbed time delay systems for all $\varepsilon_i \in (0, \infty)$. A numerical example is given to compare our new results with those in the literature.

Keywords: Lyapunov stability theorem; Singularly perturbed systems; Singular perturbation parameter; Time delay systems

1. Introduction

Many technological, environmental or societal systems have high degrees of complexity, and are usually described by high-order differential (difference) equations. While it requires a large amount of computer memory and considerable operation time to handle such large-scale systems due to the high dimensionality of these plants, the singular perturbation method provides a powerful tool to overcome these problems (Kokotovic *et al*. 1986, Kokotovic and Khalil 1986). Although singular perturbation system theory has been attracting research efforts owing to its broad range of applications, many basic issues remain unexplored. In this paper, the stability of singularly perturbed systems is focused on. The ε-bound stability problem in singularly perturbed systems has been studied extensively for decades. Thorough discussions on this problem can be found in Sen and Datta (1993), Li *et al*. (1999), Liu *et al*. (1996) and references therein. The stability problem of time-delay systems has also been explored over the years because delay is so commonly encountered in engineering systems (see, e.g., Trinh and Aldeen 1994, Hmamed 1991). In view of the above discussion, the stability problems of time-delay singularly perturbed systems are of great interest and naturally worth studying. However, there were only a few papers in the literature concerning the stability problem of time-delay singularly perturbed systems. Until now, this problem was only studied in Pan *et al*. (1996), Shao and Rowl (1995), Trinh and Aldeen (1995) and Liu (2001). Shao and Rowl (1995) presented a delay-independent sufficient condition for single time-delay in slow states. Pan *et al*. (1996) proposed a criterion in terms of the H_∞ norm to find a delay-dependent stability bound of the multiple time-delay singularly perturbed systems. These results are restricted to continuous-time cases using frequency domain techniques. For discrete-time cases, Trinh and Aldeen (1995) utilized the result of Li and Li (1992) to the robustness problem of discrete multiple time-delay singularly perturbed systems for a given $\varepsilon \in (0, \varepsilon^*)$, where ε^* is the exact ε-bound of the discrete singularly perturbed systems. A frequency domain ε-dependent stability criterion for the multiple time-delay singularly perturbed systems under the composite observer-based controller has been studied in Hsiao *et al*. (1999). In contrast, no article so far has discussed the computation of the exact ε-bound for the stability of multi-parameter singularly perturbed time-delay systems.

In this paper, stability criteria for the multi-parameter singularly perturbed time-delay systems are derived. The method is based on the Lyapunov stability theorem. The main advantage of the proposed approach is that the full-dimensional design can be directly reduced into nth lower dimensional problems without actually decomposing the system into the slow and fast

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International Journal of Systems Science
ISSN 0020–7721 print/ISSN 1464–5319 online © 2005 Taylor & Francis Group Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/00207720500156421

subsystems. Besides, the presented stabilization procedure insures the stability for an infinite ε bound.

2. System description and main results

Consider a multi-parameter singularly perturbed time-delay system.

$$\begin{aligned}\varepsilon_i \dot{x}_i(t) &= A_{ii}x_i(t) + \sum_{\substack{j\neq i\\ j=1}}^{N} A_{ij}x_j(t) + B_{ii}x_i(t-\tau)\\ &\quad + \sum_{\substack{j\neq i\\ j=1}}^{N} B_{ij}x_j(t-\tau) \quad \text{for } i=1,2,\ldots,N\end{aligned} \tag{1}$$

where $x_i \in R^{n_i}$, $A_{ii}, B_{ii} \in R^{n_i \times n_i}$, and $A_{ij}, B_{ij} \in R^{n_i \times n_j}$; the system dimension is $n = \sum_{i=1}^{N} n_i$. The small positive scales $\varepsilon_1, \ldots, \varepsilon_N$ represent time constants, inertias, masses and similar physical parameters, and $\tau > 0$ is the delay duration.

Remark 1: When $\varepsilon_i = 1$, (1) takes the form of the nominal large-scale systems with time delays as presented in Lewis and Anderson (1980) and Xu (1995). But the method in Lewis and Anderson (1980) did not allow delays in the "diagonal interactions", hence the interaction terms did not appear in Xu (1995).

For the above system, the first main objective of this paper is to find a sufficient condition that ensures the stability of multi-parameter singularly perturbed time delay systems.

To investigate the stability of system (1), the Lyapunov function candidate is chosen as

$$\begin{aligned}V_i(x_i) &= \varepsilon_i x_i^T P_i^{-1} x_i + \int_{t-\tau}^{t} x_i^T(\lambda)\\ &\quad \times \left(\sum_{\substack{j\neq i\\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right) x_i(\lambda)\, \mathrm{d}\lambda,\end{aligned} \tag{2}$$

where $P_i \in R^{n_i \times n_i}$ is the unique real symmetric positive-definite matrix.

Lemma 1 (Zhou and Khargonekar 1988)**:** *For any matrices X and Y with appropriate dimensions,*

$$X^T Y + Y^T X \le \gamma X^T X + \frac{1}{\gamma} Y^T Y \tag{3}$$

for any constant $\gamma > 0$.

Theorem 1: *The system* (1) *is asymptotically stable independent of delay* (*i.o.d.*) *for* $\varepsilon_i \in (0, \infty)$, *if there exists a matrix* $P_i > 0$ *such that*

$$\begin{bmatrix} P_i A_{ii}^T + A_{ii} P_i + (2N-1)I & P_i C_i \\ C_i^T P_i & -I \end{bmatrix} < 0 \tag{4}$$

holds for $i = 1, 2, \ldots, N$, *where*

$$C_i C_i^T = \sum_{\substack{j\neq i\\ j=1}}^{N} (A_{ji}^T A_{ji} + B_{ji}^T B_{ji}) + B_{ii}^T B_{ii}.$$

Proof: Let the Lyapunov function for the overall system be

$$\begin{aligned}V &= \sum_{i=1}^{N} V_i\\ &= \sum_{i=1}^{N} \left(\varepsilon_i x_i^T P_i^{-1} x_i + \int_{t-\tau}^{t} x_i^T(\lambda) \right.\\ &\quad \left. \times \left(\sum_{\substack{j\neq i\\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right) x_i(\lambda)\, \mathrm{d}\lambda \right).\end{aligned} \tag{5}$$

The derivative of Lyapunov function becomes

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N} \dot{V}_i(x_i)\\ &= \sum_{i=1}^{N} \left\{ \varepsilon_i \dot{x}_i^T P_i^{-1} x_i + \varepsilon_i x_i^T P_i^{-1} \dot{x}_i \right.\\ &\quad + x_i^T(t) \left(\sum_{\substack{j\neq i\\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right) x_i(t)\\ &\quad \left. - x_i^T(t-\tau) \left(\sum_{\substack{j\neq i\\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right) x_i(t-\tau) \right\}\\ &= \sum_{i=1}^{N} \left\{ \left[A_{ii}x_i(t) + \sum_{\substack{j\neq i\\ j=1}}^{N} A_{ij}x_j(t) + B_{ii}x_i(t-\tau) \right. \right.\\ &\quad \left. + \sum_{\substack{j\neq i\\ j=1}}^{N} B_{ij}x_j(t-\tau) \right]^T P_i^{-1} x_i\end{aligned}$$

$$+ x_i^T P_i^{-1} \left[A_{ii} x_i(t) + \sum_{\substack{j \neq i \\ j=1}}^{N} A_{ij} x_j(t) + B_{ii} x_i(t-\tau) \right.$$

$$\left. + \sum_{\substack{j \neq i \\ j=1}}^{N} B_{ij} x_j(t-\tau) \right] + x_i^T(t) \left(\sum_{\substack{j \neq i \\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right) x_i(t)$$

$$\left. - x_i^T(t-\tau) \left(\sum_{\substack{j \neq i \\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right) x_i(t-\tau) \right\}$$

$$\leq \sum_{i=1}^{N} x_i^T \left[A_{ii}^T P_i^{-1} + P_i^{-1} A_{ii} + (2N-1) P_i^{-2} \right] x_i$$

$$+ \sum_{\substack{j \neq i \\ j=1}}^{N} x_j^T A_{ij}^T A_{ij} x_j + \sum_{\substack{j \neq i \\ j=1}}^{N} x_j^T(t-\tau) B_{ij}^T B_{ij} x_j(t-\tau)$$

$$+ x_i^T(t-\tau) B_{ii}^T B_{ii} x_i(t-\tau) + x_i^T(t) \left(\sum_{\substack{j \neq i \\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right)$$

$$\times x_i(t) - x_i^T(t-\tau) \left(\sum_{\substack{j \neq i \\ j=1}}^{N} B_{ji}^T B_{ji} + B_{ii}^T B_{ii} \right) x_i(t-\tau)$$

(by Lemma 1 and Let $\gamma = 1$)

$$= \sum_{i=1}^{N} \left\{ x_i^T \left[A_{ii}^T P_i^{-1} + P_i^{-1} A_{ii} + (2N-1) P_i^{-2} \right. \right.$$

$$\left. \left. + \sum_{\substack{j \neq i \\ j=1}}^{N} (A_{ji}^T A_{ji} + B_{ji}^T B_{ji}) + B_{ii}^T B_{ii} \right] x_i \right\}. \tag{6}$$

Equation (6) is negative if

$$A_{ii}^T P_i^{-1} + P_i^{-1} A_{ii} + (2N-1) P_i^{-2} + \sum_{\substack{j \neq i \\ j=1}}^{N} (A_{ji}^T A_{ji} + B_{ji}^T B_{ji}) + B_{ii}^T B_{ii} < 0. \tag{7}$$

Pre- and post-multiplying (7) byP_i, then

$$P_i A_{ii}^T + A_{ii} P_i + (2N-1) I + P_i C_i C_i^T P_i < 0. \tag{8}$$

If inequality (8) holds, then $\dot{V} < 0$, and from Schur complement, it is easy to find that the matrix inequality (8) is equivalent to the LMI (4). □

Remark 2: In the light of Theorem 1, it is seen that interconnection cannot be too strong. If there is a large $A_{ji}^T A_{ji}$ and/or a large $B_{ji}^T B_{ji}$, one can decouple ε_j to $\bar{\varepsilon}_j / \hat{\varepsilon}_j$, and choose $\hat{\varepsilon}_j$ such that $\hat{\varepsilon}_j^2 A_{ji}^T A_{ji}$ and/or $\hat{\varepsilon}_j^2 B_{ji}^T B_{ji}$ are as small as possible. Here $\hat{\varepsilon}_j$ is an auxiliary parameter. The aim of introducing $\hat{\varepsilon}_j$ is to provide an additional design freedom since $\bar{\varepsilon}_j \in (0, \infty)$ implies $\varepsilon_j \in (0, \infty)$.

In addition to Theorem 1, another delay-independent criterion that may be helpful for stability testing for the system (1) is given below.

Consider the linear time-delay systems

$$\dot{x}(t) = Ax(t) + Bx(t-\tau), \tag{9}$$

where $x \in R^n$, A and B are matrices of appropriate dimensions; τ is the delay duration.

Concerning the stability of (9), a preliminary result is given below.

Lemma (Brierley *et al*. 1982) **2:** *Stability of system* (3) *implies stability of the following system*

$$\dot{w}(t) = (A + zB) w(t) \quad \forall |z| = 1, \tag{10}$$

and vice versa.

Remark 3: In the light of Lemma 2, it is obvious that the asymptotic stability of the system (1) is equivalent to that of the system

$$\varepsilon_i \dot{y}_i(t) = (A_{ii} + zB_{ii}) y_i(t) + \sum_{\substack{j \neq i \\ j=1}}^{N} (A_{ij} + zB_{ij}) y_j(t) \tag{11}$$

where $y_i \in R^{n_i}$ and $\forall |z| = 1$.

To investigate the stability of system (13), the Lyapunov function candidate is chosen as

$$V_i(y_i) = \varepsilon_i y_i^T(t) P_i^{-1} y_i(t) \tag{12}$$

where $P_i \in C^{n_i \times n_i}$ are unique complex symmetric positive-definite matrices.

In the following theorem, the above concepts and complex Lyapunov function are used to derive a new sufficient condition for the stability of the multi-parameter singularly perturbed time delay systems.

Theorem 2: *The system* (1) *is asymptotically stable independent of delay for all* $\varepsilon_i \in (0, \infty)$, *if there exists a matrix* $P_i > 0$ *such that*

$$\begin{bmatrix} P_i(A_i + zB_i)^* + (A_{ii} + zB_i) P_i + (N-1) I & P_i D_i \\ D_i^T P_i & -I \end{bmatrix} < 0 \tag{13}$$

holds for $i = 1, 2, \ldots, N$, *where*

$$D_i D_i^T = \sum_{\substack{j \neq i \\ j=1}}^{N} (A_{ji} + zB_{ji})^T (A_{ji} + zB_{ji}).$$

Proof: Let the Lyapunov function for the overall system be chosen as

$$\begin{aligned} V(y) &= \sum_{i=1}^{N} V_i(y_i) \\ &= \sum_{i=1}^{N} \varepsilon_i y_i^T(t) P_i^{-1} y_i(t) \end{aligned} \tag{14}$$

The derivative of Lyapunov function becomes

$$\begin{aligned} \dot{V}(y) &= \sum_{i=1}^{N} \dot{V}_i(y_i) = \sum_{i=1}^{N} (\varepsilon_i \dot{y}_i^T P_i^{-1} y_i + \varepsilon_i y_i^T P_i^{-1} \dot{y}_i) \\ &= \sum_{i=1}^{N} \left\{ \left[(A_{ii} + zB_{ii}) y_i + \sum_{\substack{j \neq i \\ j=1}}^{N} (A_{ij} + zB_{ij}) y_i^T \right] P_i^{-1} y_i \right. \\ &\quad \left. + y_i^T P_i^{-1} \left[(A_{ii} + zB_{ii}) y_i + \sum_{\substack{j \neq i \\ j=1}}^{N} (A_{ij} + zB_{ij}) y_j \right] \right\} \\ &\le \sum_{i=1}^{N} \left\{ y_i^T (A_{ii}^T P_i^{-1} + P_i^{-1} A_{ii} + z^* B_{ii}^T P_i^{-1} + zP_i^{-1} B_{ii}) y_i \right. \\ &\quad \left. + \sum_{\substack{j \neq i \\ j=1}}^{N} \left[y_i^T P_i^{-2} y_i + y_j^T (A_{ij}^T + z^* B_{ij}^T)(A_{ij} + zB_{ij}) y_j \right] \right\} \\ &\le \sum_{i=1}^{N} \left\{ y_i^T (A_{ii}^T P_i^{-1} + P_i^{-1} A_{ii} + z^* B_{ii}^T P_i^{-1} + zP_i^{-1} B_{ii} \right. \\ &\quad \left. + (N-1) P_i^{-2} + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left[(A_{ji}^T + z^* B_{ji}^T)(A_{ji} + zB_{ji}) \right] \right\} y_i. \end{aligned} \tag{15}$$

Equation (15) is negative if

$$\begin{aligned} A_{ii}^T P_i^{-1} + P_i^{-1} A_{ii} + z^* B_{ii}^T P_i^{-1} + zP_i^{-1} B_{ii} + (N-1) P_i^{-2} \\ + \sum_{\substack{j \neq i \\ j=1}}^{N} (A_{ji}^T + z^* B_{ji}^T)(A_{ji} + zB_{ji}) < 0. \end{aligned} \tag{16}$$

Pre- and post-multiplying (16) by P_i, then

$$\begin{aligned} P_i A_{ii}^T + A_{ii} P_i + z^* P_i B_{ii}^T + zB_{ii} P_i + (N-1) I \\ + \sum_{\substack{j \neq i \\ j=1}}^{N} P_i (A_{ji}^T + z^* B_{ji}^T)(A_{ji} + zB_{ji}) P_i < 0. \end{aligned} \tag{17}$$

If inequality (17) holds, then $\dot{V} < 0$, and from Schur complement, it is easy to find that the matrix inequality (17) is equivalent to the complex-valued LMI (13).

3. An example

Example: Consider the following multiparameter singularly perturbed time delay systems.

$$\begin{aligned} \varepsilon_1 \dot{x}_1(t) &= \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix} x_2(t) \\ &\quad + \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix} x_1(t-\tau) + \begin{bmatrix} 0.1 & 0 \\ -1 & 0.3 \end{bmatrix} x_2(t-\tau) \\ \varepsilon_2 \dot{x}_2(t) &= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} x_1(t) + \begin{bmatrix} -20 & 0 \\ -20 & -30 \end{bmatrix} x_2(t) \\ &\quad + \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} x_1(t-\tau) + \begin{bmatrix} 1 & 0 \\ 5 & 10 \end{bmatrix} x_2(t-\tau). \end{aligned} \tag{18}$$

The main objective here is to analyze the above system's stability. First, using those reported in the literature (Mori *et al.* 1981, Hmamed 1991, Shyu and Yan 1993, Lee *et al.* 1994, Xu and Liu 1994, Huang *et al.* 1995, Xu 1995, Yan *et al.* 2001) and Theorems 1 and 2, stability test fails when $\varepsilon_1 = \varepsilon_2 = 1$. Thus, the proposed method in Mori *et al.* (1981), Hmamed (1991), Shyu and Yan (1993), Lee *et al.* (1994), Xu and Liu (1994), Huang *et al.* (1995), Xu (1995), Yan *et al.* (2001) and Theorems 1 and 2 yield no conclusion. Furthermore, from remark 2, we can choose $\hat{\varepsilon}_1 = 1$ and $\hat{\varepsilon}_2 = 0.1$; hence, an alternative form can represent the above system as follows:

$$\begin{aligned} \bar{\varepsilon}_1 \dot{x}_1(t) &= \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix} x_2(t) \\ &\quad + \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix} x_1(t-\tau) + \begin{bmatrix} 0.1 & 0 \\ -1 & 0.3 \end{bmatrix} x_2(t-\tau) \\ \bar{\varepsilon}_2 \dot{x}_2(t) &= \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.2 \end{bmatrix} x_1(t) + \begin{bmatrix} -2 & 0 \\ -2 & -3 \end{bmatrix} x_2(t) \\ &\quad + \begin{bmatrix} 0 & 0.1 \\ 0 & 0.1 \end{bmatrix} x_1(t-\tau) + \begin{bmatrix} 0.1 & 0 \\ 0.5 & 1 \end{bmatrix} x_2(t-\tau). \end{aligned} \tag{19}$$

Using those reported in the literature (Mori *et al.* 1981, Hmamed 1991, Shyu and Yan 1993, Lee *et al.* 1994, Xu and Liu 1994, Huang *et al.* 1995, Xu 1995, Yan *et al.* 2001) and Theorem 1, (19) fails the stability test. While by Yan *et al.* (2001) and Theorem 2 for which

$$P_1 = \begin{bmatrix} 1.2126 & -1.2128 \\ -1.2128 & 2.3412 \end{bmatrix}$$

and

$$P_2 = \begin{bmatrix} 0.9526 & -0.5267 - 0.0081i \\ -0.5268 + 0.0081i & 1.1853 \end{bmatrix}$$

in the LMI (13), the stability of the system (19) is guaranteed. Note that the stability of the system (19) cannot imply to the stability of the system (18) by Yan *et al.* (2001). However, it is concluded that the stability of the system (19) implies the stability of the system (18). Hence, the system (18) is asymptotically stable independent of delay for all $\varepsilon_i \in (0, \infty)$.

4. Conclusion

For a class of multi-parameter singularly perturbed time delay systems, this paper has presented two new sufficient conditions of stability test for all $\varepsilon_i \in (0, \infty)$ independent of time-delay. The result gives an explicit representation and simple computation in the stability testing. This paper has three major contributions: First, there are infinite objects that can be tested by decomposing singularly perturbed parameters. Second, the full-dimensional process can be separated into nth order subsystems. Finally, the system is stable for all $\varepsilon_i \in (0, \infty)$ and independent of time-delay.

Acknowledgment

This work is supported by the National Science Council, Taiwan, Republic of China, under grand number NSC 91-2213-E-218-016.

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