Chap 5

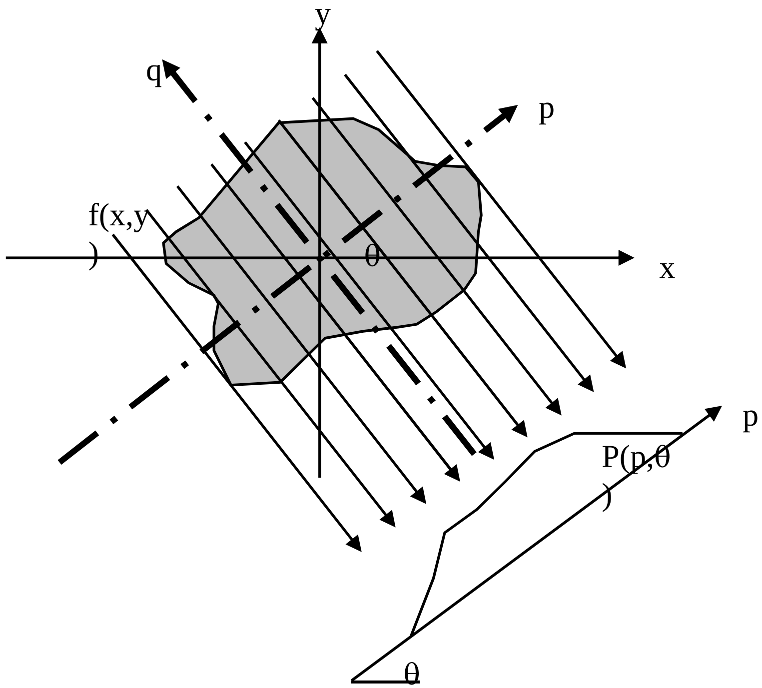
Image Reconstruction

Radon Transform

The Radon transform of an object f(x,y) is expressed as the projection $J_\theta(p)$

$$R\{f(x,y)\} = J_\theta(p) = \int_{-\infty}^{\infty} f(p\cos\theta - q\sin\theta, p\sin\theta + q\cos\theta)dq$$

Line integral projection P(p,θ) of the two-dimensional Radon transform.



Central Slice Theorem

- The Central Slice Theorem(the projection theorem)
  - provides a relationship between the Fourier transform of the object function and the Fourier transform of its Radon transform or projection.

$$F\{R\{f(x,y)\}\} = F\{J_\theta(p)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(p\cos\theta - q\sin\theta, p\sin\theta + q\cos\theta)e^{-j2\pi\omega p}dqdp$$

It can also be expressed as

$$S_\theta(w) = \int_{-\infty}^{\infty} J_\theta(p)e^{-j2\pi\omega p}dp = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi\omega(x\cos\theta + y\sin\theta)}dxdy = F(w,\theta)$$

where ω represents the frequency component in the Fourier domain

Central Slice Theorem

The frequency domain of the Fourier transform F(u,v) with the Fourier transforms $S_\theta(w)$ of individual projections $J_\theta(p)$

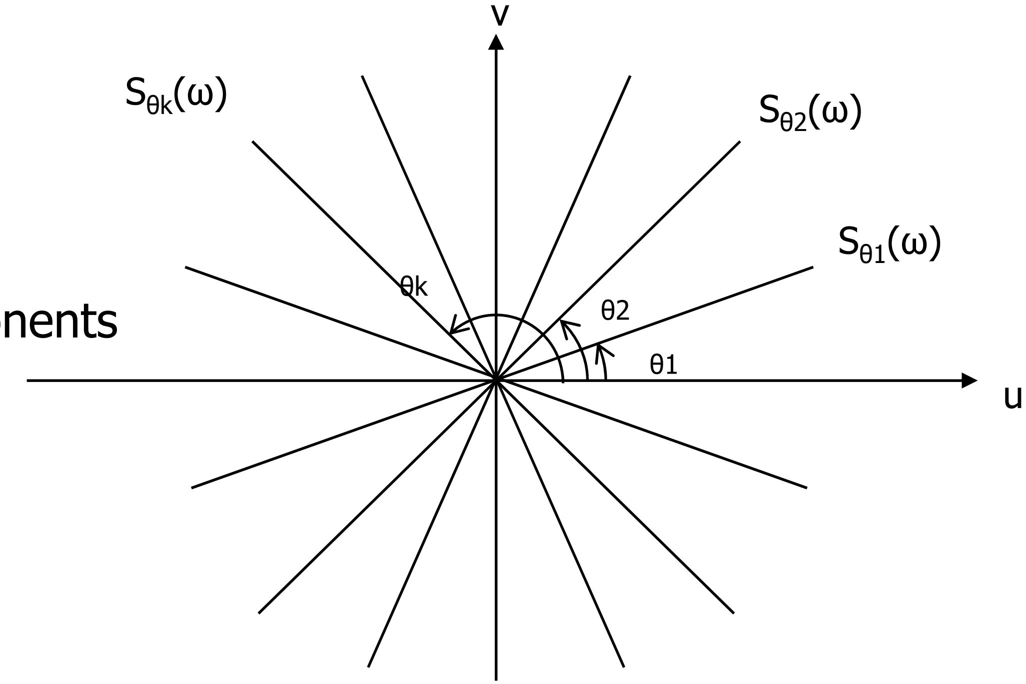
$$S_\theta(w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi\omega(x\cos\theta + y\sin\theta)}dxdy = F(w,\theta) = F(u,v)$$

where

$$u = \omega\cos\theta$$

$$v = \omega\sin\theta$$

u and v represent frequency components along the x- and y-directions in a rectangular coordinate system



Back projection Reconstruction Method

The modified projection

$$J_{\theta}^{*}(p') = F^{-1}\{|w| S_{\theta}(\omega)\}$$
$$= F^{-1}\{|w|\} \otimes J_{\theta}(p)$$

$$J_{\theta}^{*}(p') = \int_{-\infty}^{\infty} J_{\theta}(p') h(p - p') dp'$$

$$\hat{f}(x, y) = \frac{\pi}{L} \sum_{i=1}^{L} J_{\theta_i}(p')$$

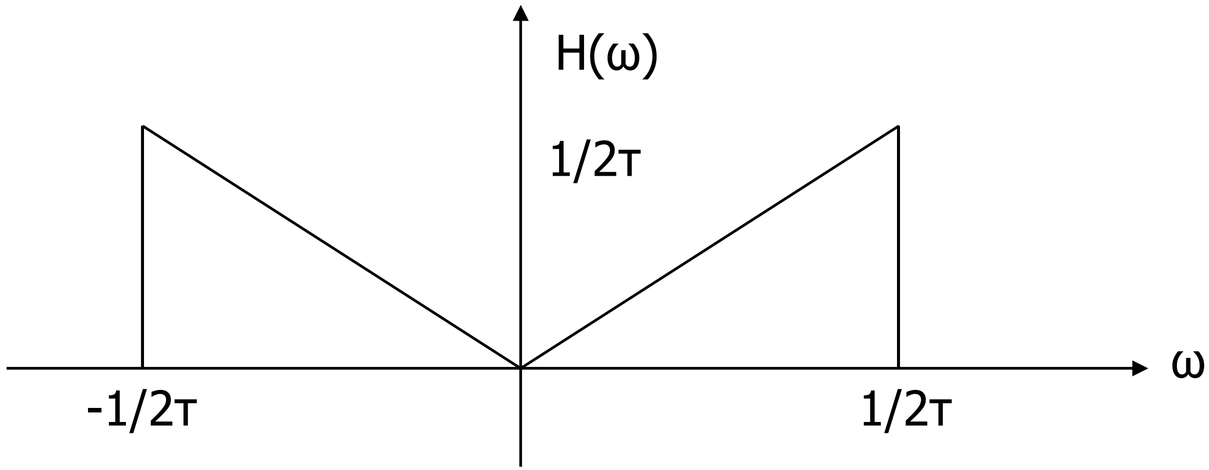
where L is the total number of projections acquired during the imaging process at viewing angles

$$\theta_i \text{ ; for i} = 1, .., \text{L}$$

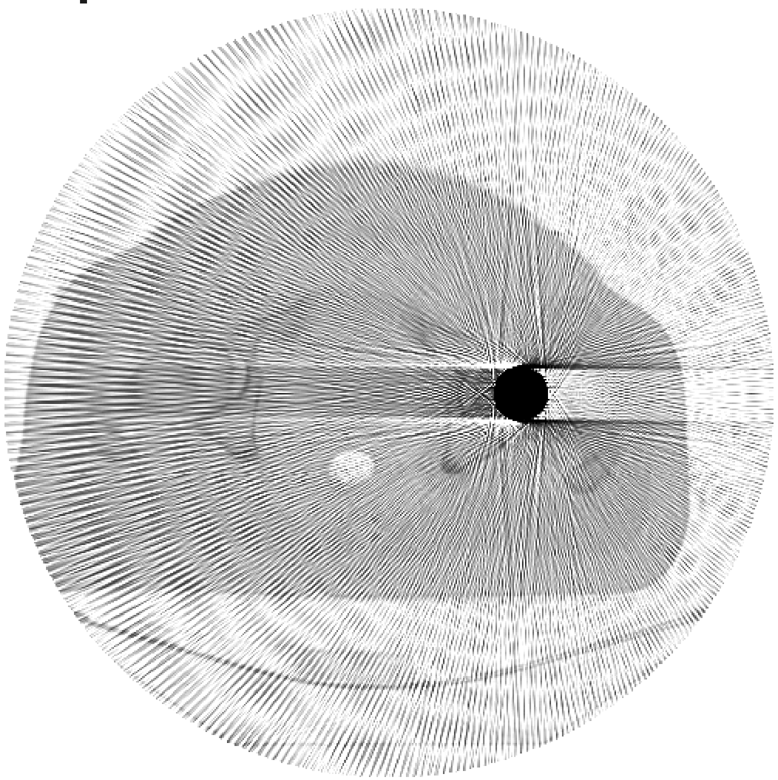
Back projection Reconstruction Method

- Define the filter function $h_{R\text{-}L}(p)$ in spatial domain and the fourier transform $H_{R\text{-}L}(\omega)$

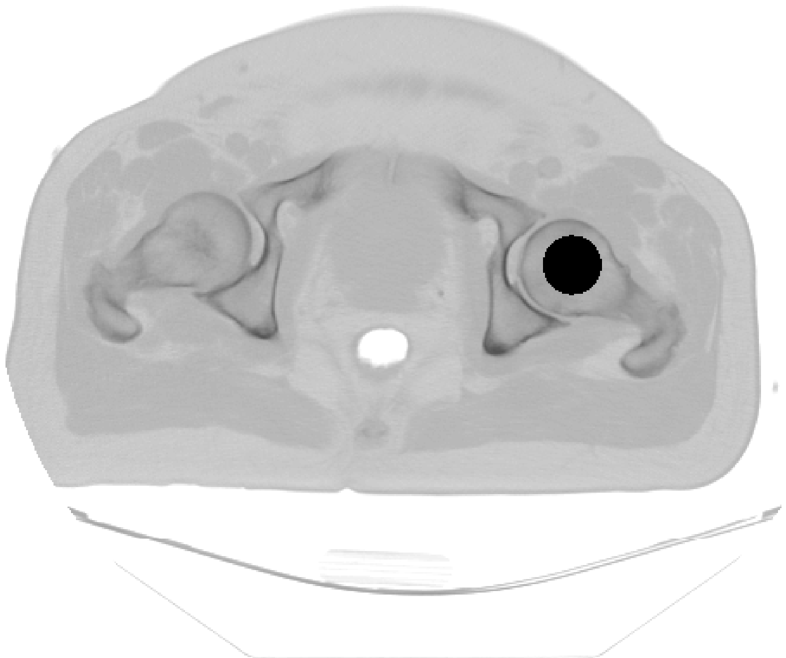
$$H_{R-L}(\omega) = \begin{cases} |\omega| & if\ |\omega| \le \Omega \\ 0 & otherwise \end{cases}$$



Back projection Reconstruction



Back Projection



Modified Filtered **Back Projection**

ML-EM Image Reconstruction in ECT

Let us assume that the object to be reconstructed has an emission density function $\lambda(x, y, z) = \vec{\lambda} = [\lambda_1, \lambda_2, ...\lambda_B]^t$ with a Poisson process over a matrix of B pixels. The emitted photons (in case of SPECT) or photon pairs (in case of PET) are detected by the detectors with the measurement vector $\vec{J} = [J_1, J_2, ..., J_D]^t$ with D measurements. The problem is then to estimate $\lambda(b); b = 1, ..., B$ from the measurement vector.

Each emission in box b is detected by the detector d (SPECT) or the detector tube d (PET) with probability $p(b, d) = P$(detection in d | photon emitted in b). The transition matrix $p(b, d)$ is derived from the geometry of the detector array and the reconstruction space.

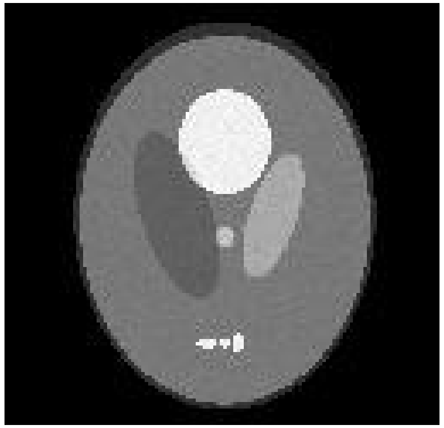
Let $j(b, d)$ denotes the number of emissions in box b detected in the detector or detector tube d are independent Poisson variables with the expected value $E[j(b, d)] = \lambda(b, d) = \lambda(b) p(b, d)$.

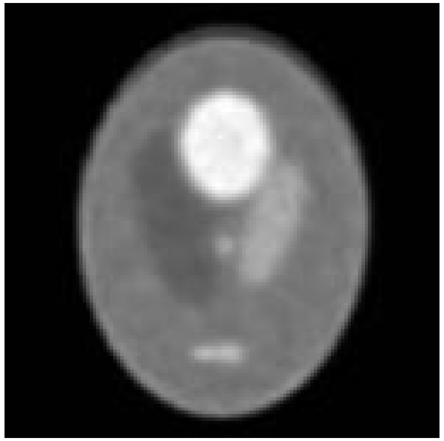
$$L(\lambda) = P(\vec{j} \mid \vec{\lambda}) = \sum_{A} \prod_{\substack{b=1,\ldots,B \\ d=1,\ldots,D}} e^{-\lambda(b,d)} \frac{\lambda(b,d)^{j(b,d)}}{j(b,d)!}$$

$$\hat{\lambda}^{new}(b) = \hat{\lambda}^{old}(b) \sum_{d=1}^{D} \frac{j(d)\, p(b,d)}{\sum_{b'=1}^{B} \hat{\lambda}^{old}(b')\, p(b',d)}; \qquad b = 1,\ldots,B$$

Reconstruction in PET

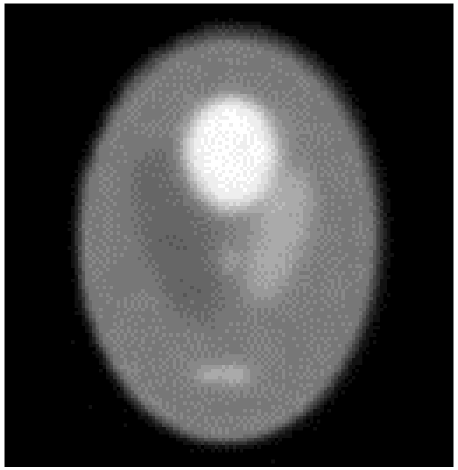
Phantom





Wmulti-Grid
EM

ML-EM



BackProj

Reconstruction in MRI

$$S(\vec{\omega}) = \vec{M}_0 \iiint f\,(x, y, z) e^{-i(\omega_x x + \omega_y y + \omega_z z)} dxdydz$$

Fourier Transform Reconstruction Method