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A study of an integrated inventory model for imperfect production system with backorders

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Abstract

In recent years, companies have found that there are substantial benefit and competitive advantage from establishing a long-term sole-supplier relationship with supplier. In the integrated inventory model practices characterized by a sole-supplier base whose firm is located close to the buyer's plant and makes frequent deliveries is considered a long-term partner with the buying company. The characteristics of the integrated inventory model are consistent high quality, small lot sizes, frequent delivery, short lead time, and close supplier ties. This paper presents an integrated inventory model with backorders. That is, it is assumed that shortage during the lead time is permitted, the lead time can be reduced at added cost. The purpose of this research is to develop an integrated inventory model for minimizing the total joint annual costs incurred by the vendor and the purchaser. An algorithmic procedure is established to find the optimal order quantity, lead time and number of deliveries simultaneously for the case of normally distributed lead time demand.

Keywords: lead time, integrated inventory model, imperfect production system

1. Introduction

In the dynamic, competitive environment, supply chain management has emerged as a popular production and logistics strategy for many contemporary firms, and the just-in-time (JIT) purchasing plays a crucial role in such supply chain environments. Companies are using JIT purchasing to gain and maintain a competitive advantage. The benefits of JIT purchasing include small lot sizes, frequent deliveries, consistent high quality, reduction in lead times, decrease in inventory levels, lower setup cost and ordering cost, and close supplier ties. In recent years, companies have found that there are substantial benefits from establishing a long-term sole-supplier relationship with supplier (Martinich 1997). In the JIT environment, a close cooperation exists between supplier and purchaser to solve problems together, and thus maintains stable, long-term relationships.

Goyal (1977) suggested a joint economic lot size model where the objective was to minimize the total relevant costs for both the vendor and the buyer. Banerjee (1986) presented a joint economic-lot-size model where a vendor produced to order for a purchaser on a lot-for-lot basis under deterministic conditions. Goyal (1988) generalized the Banerjee model (1986) by relaxing the assumption of the lot-for-lot policy of the vendor. Lu (1995) presented a model for the one-vendor one-buyer problems, and developed a heuristic approach for the one-vendor multi-buyer case. The model is an improvement over Banerjee (1986) and Goyal (1988) models. Ha and Kim (1997) addressed an integrated lot-splitting model of facilitating multiple shipments in small lots and compared it with some existing approaches in a JIT environment.

Traditionally, the lead time of inventory model is hypothesized as known or with certain probability distribution, which therefore is not subject to control. But in many practical situations, lead time can be reduced by an additional crashing cost. That is, it is controllable. In fact, as pointed out in Tersine (1982), lead time consists mainly of the following components: order preparation, order transit, supplier lead time, and delivery time. Recently, lead time reduction has received a lot of interest by several researchers (Liao and Shyu 1991, Ben-Daya and Daouf 1994, Ouyang *et al*. 1996, Ouyang and Wu 1997, Moon and Choi 1998, Ouyang *et al.* 2002).

In a recent paper, Huang (2002) developed an integrated inventory model for items with imperfect quality. Previous researches (Goyal 1977, Banerjee 1986, Goyal 1988, LU 1995, Ha and Kim 1997, Khan and Sarker 2002) on the integrated inventory model most focused on the production shipment schedule in terms of the number and size of batches transferred between both parties under deterministic demand. However, defective items are often and inevitably produced in real production systems. These defective items must be rejected, repaired, reworked, or, if they have reached the customer, refunded. In all cases, substantial costs are incurred

(Ouyang *et al.* 2002)**.** Porteus (1986) and Rosenblatt and Lee (1986) first presented the significant relationship between quality imperfection and lot size. Following Porteus (1986), there are abundant quality improvement related literatures, for example, see Keller and Noori (1988), Hong and Hayya (1995), Ouyang and Chang (2000), and Ouyang *et al.* (2002).

Many studies on the related issues have focused on the benefit only from a single party's viewpoint; however, consideration of the dyadic relationship between the vendor and purchaser is necessary for a successful implementation of JIT purchasing. When entering a long-term relationship, one partner's gain exceeds the other partner's loss and the net benefit can be shared by both parties in some equitable fashion (Goyal and Gupta 1989).

In this paper, the task is to extend Pan and Yang (2002) inventory model. Pan and Yang (2002) presented an integrated supplier-purchaser model focused on the benefit from lead time reduction while excluding quality-related issue. This paper serves as a pioneering work on investigating the effects of lead time reduction, frequent delivery and quality-related cost on the JIT purchasing inventory model. An algorithmic procedure is established to find the optimal order quantity, lead time and number of deliveries.

2. Notations and assumptions

To establish the proposed model, the following notations are used:

D = average demand per year;
P = production rate;
Q = contract quantity;
A = purchaser's ordering cost per order;
S = vendor's setup cost per setup;
L = length of lead time;
h_s = unit holding cost paid by the vendor;
h_b = unit holding cost paid by the purchaser;

The assumptions made in the paper are as follow:

(1) Product is manufactured with a finite production rate P, and $P > D$.
(2) The demand X during lead time L follows a normal distribution with mean DL and standard deviation σ.
(3) The reorder point (ROP) equals the sum of the expected demand during lead time and the safety stock, that is, the reorder point $R = \mu L + k\sigma\sqrt{L}$, where k is known as the safety factor.
(4) Inventory is continuously reviewed and replenished.
(5) The lead time L has n mutually independent components and these components are crashed one at a time starting with the one of least crashing cost per unit time, and so on.

3. Model formulation

Since A is the ordering cost per order, the expected ordering cost per year is given by $(D/Q)A$. The expected holding cost per year is $h_b(Q/2N + R - DL)$. The expected shortage cost per year is DNF/Q. Since the unit backorder cost is π, the expected shortage cost per year is given by $(\pi DN/Q)E(X - R)^+$. The total expected annual cost for the purchaser is given by:

$$TC_b(Q, L, N) = \frac{AD}{Q} + h_b\left(\frac{Q}{2N} + R - DL\right) + \frac{\pi DN}{Q}E(X-R)^+ + \frac{DN}{Q}F + \frac{DN}{Q}R(L) \tag{1}$$

where N is an integer representing the number of shipments that the item is delivered to the purchaser, and a_i, b_i, c_i are the minimum duration, normal duration and crashing cost per unit time, respectively, of the ith component of lead time. Let $\sum_{i=1}^{n} a_i \le L \le \sum_{i=1}^{n} b_i$, and L_i be the length of ith component of lead time crashed to its minimum duration. Then L_i can be expressed as $L_i = \sum_{i=1}^{n} b_i - \sum_{j=1}^{i}(b_j - a_j)$, $i = 1, 2, \ldots, n$. Also let $R(L)$ denote the lead time crashing cost per cycle for a given $L \in [L_i, L_{i-1}]$, and $R(L) = c_i[L_{i-1} - L] + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

In order to include the very truth of imperfect production process, consider the assumption made in the model propose by Porteus (1986). The integrated inventory model is designed for vendor's production situations in which once an order is placed, the production begins and a constant number of units are added to inventory each day until the production run has been completed. The vendor produces the item in the quantity of Q with a given probability of θ that the process can go out-of-control. Porteus (1986) suggested the expected number of defective items in a run of size Q can be evaluated as $Q^2\theta/2$. Suppose s is the cost of replacing a defective unit, and the production quantity for the supplier in a lot of Q, then its expected defective cost per year is given by $sQD\theta/2$. For the vendor's inventory model, its total expected annual cost can be represented by:

$$TC_s(Q, N) = \frac{DS}{Q} + \frac{Qh_s}{2N}\left[(2-N)\frac{D}{P} + N - 1\right] + \frac{sDQ\theta}{2} \tag{2}$$

If the purchaser's order quantity and the vendor's lot size is Q, then the joint total expected annual cost is given by:

$$\begin{aligned} JTC(Q, L, N) &= TC_b(Q, N, L) + TC_s(Q, N) \\ &= \frac{D}{Q}[A + S + N(\pi E(X-R)^+ + F + R(L))] + h_b\left(\frac{Q}{2N} + k\sigma\sqrt{L}\right) + \frac{Q}{2N}h_s\left[(2-N)\frac{D}{P} + N - 1\right] + \frac{sDQ\theta}{2} \\ &= \frac{D}{Q}[A + S + N(\pi E(X-R)^+ + F + R(L))] + \frac{Q}{2N}\left[h_b + h_s\left((2-N)\frac{D}{P} + N - 1\right) + sD\theta N\right] + h_b k\sigma\sqrt{L} \end{aligned} \tag{3}$$

As shortage is allowed, Ouyang and Chang (2000) pointed out that the expected demand shortage at the end of the cycle is $\sigma\sqrt{L}\ \psi(k)$, where $\psi(k) = \phi(k) - k[1 - \Phi(k)]$, and ϕ and Φ denote the standard normal probability density function and distribution function, respectively. Since the unit backorder cost is π, the expected shortage cost per year is given by $\pi(D/Q)\sigma\sqrt{L}\ \psi(k)$. The expected annual total cost of an integrated inventory model with normally distributed lead time demand for minimizing the sum of the ordering cost, holding cost, stockout cost, transportation cost and crashing cost can be expressed as:

$$JTC(Q, N, L) = \frac{D}{Q}[A + S + N(\pi\sigma\sqrt{L}\ \Psi(k) + F + R(L)] + \frac{Q}{2N}\left[h_b + h_s\left((2-N)\frac{D}{P} + N - 1\right) + sD\theta N\right] + h_b k\sigma\sqrt{L} \tag{4}$$

Taking the partial derivatives of $JTC(Q, N, L)$ with respect to Q and L in each time interval (L_i, L_{i-1}), and equating them to zero, we obtain:

$$\frac{\partial JTC(Q,N,L)}{\partial Q} = -\frac{D}{Q^2}[A + S + N(\pi(\sqrt{L}\ \Psi(k) + F + R(L))) + \frac{1}{2N}\left[h_b + h_s\left((2-N)\frac{D}{P} + N - 1\right)\right] + \frac{sD\theta}{2} \tag{5}$$

$$\frac{\partial JTC(Q,N,L)}{\partial Q} = \frac{DN}{Q}\left(\frac{\pi\sigma\psi(k)}{2\sqrt{L}} - C_i\right) + \frac{h_b k\sigma}{2\sqrt{L}} \tag{6}$$

However, for fixed Q, $JTC(Q, N, L)$ is concave in $L \in (L_i, L_{i-1})$, because

$$\frac{\partial^2 JTC}{\partial L^2} = -\frac{DN}{4Q}\pi\sigma\Psi(k)\, L^{-\frac{3}{2}} - \frac{1}{4}h_b K L^{-\frac{3}{2}} < 0 \tag{7}$$

Hence, for fixed $L \in (L_i, L_{i-1})$, $JTC(Q, N, L)$ is convex in Q, since

$$\frac{\partial^2 JTC(Q,N,L)}{\partial Q^2} = \frac{2D}{Q^3}[A + S + N(\pi\sigma\sqrt{L}\ \Psi(k) + F + R(L))] > 0 \tag{8}$$

Therefore, for fixed Q, the minimum joint total expected annual cost will occur at the end points of the interval. From (5), we have

$$Q = \left[\frac{2DN[A + S + N(\pi\sigma\sqrt{L}\psi(k) + F + R(L))]}{h_b + h_s[(2-N)\dfrac{D}{P} + N - 1] + sDN\theta} \right]^{\frac{1}{2}} \quad (9)$$

The optimal value of $N = N^*$ is obtained when $JTC(N^* - 1) \geq JTC(N^*) \leq JTC(N^* + 1)$. In (4) the only unknown variable is N. Lel $N = 1,2,3,\ldots$ and choose the one for which equation (4) is minimized. Hence, for fixed $L \in [L_i, L_{i-1}]$, one can find the optimal values of Q^*, N^* and L^* such that the joint total expected annual cost reaches a minimum.

4. An illustrative example

To illustrate the above solution procedure, consider an inventory system with following data : D = 12,000 unit/year, P = 48,000 unit/year, A = \$25/order, S = \$500/setup, F = \$25/shipment, h_b =\$12/unit/year, h_s = \$10/unit/year, k = 2.33, σ = 15 unit/week, π = \$10 per unit, θ = 0.0002, s = \$3 per unit, and the lead time has three components as shown in Table 1.

Applying the proposed solution procedure yields optimal order quantity Q = 929 units, optimal number of delivery N = 3, and optimal lead time L = 42 days with the joint total expected annual cost of \$16,845.8. The associated data of the solution procedure are summarized in Table 2.

Table 1. Lead time data for the illustrative example

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.2
2	20	6	0.4
3	16	9	0.8

Table 2. Summary of the solution procedure for the illustrative example

N^*	L_N^*	Q_N^*	Purchaser's cost	Vender's cost	JTC^*
1	3	790	\$6,451.80	\$11,426.44	\$17,878.24
2	4	885	\$4,765.82	\$12,178.16	\$16,943.98
3	6	929	\$4,333.26	\$12,512.54	\$16,845.80*
4	6	967	\$4,229.13	\$12,707.83	\$16,936.96
5	6	999	\$4,270.06	\$12,849.16	\$17,119.22

5. Conclusions

The proposed model is shown to provide a lower total inventory cost, more frequent deliveries with smaller lot sizes and shorter lead time. The characteristics of JIT systems are shortening lead time, making frequent deliveries with small lot sizes and close supplier ties. The proposed model shows that it is consistent with the JIT purchasing philosophy and is more attractive than the traditional integrated inventory model.

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允許欠撥缺貨情況下考慮不完美品質生產系統之整合性存貨模式研究

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摘要

近年來，企業與單一供應商建立良好的長期關係，並從中獲取實質利益與競爭優勢。整合性存貨模式優點為買方與賣方密切合作，長期採購合約約定賣方少量多次的小批量運送產品，進而達到高品質、低存貨、交貨頻繁、短前置時間與緊密合作的關係。本文是研究當欠撥缺貨現象發生時之整合性存貨模式，假設允許缺貨現象發生，前置時間可以經由付出額外的趕工成本而改變。本研究旨在針對前置時間內需求量服從常態分配的前提下，使得買方與賣方所發生總存貨成本最小化，而提出一個整合性存貨模式，並發展模式最適解的演算法，同時得出訂購量、前置時間與交貨次數最適解。

關鍵字：前置時間、整合性存貨模式、不完美品質生產系統