Frame Synchronization of OFDM Systems in Frequency Selective Fading Channels

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Fractional Frequency Offset Estimation

\[ c_i = \tilde{s}^*(t_i) \cdot \tilde{s}(t_i + T_u) \]  
\[ = |A(t_i)|^2 \cdot \{ e^{-j2\pi\Delta f t_i} \cdot e^{j2\pi\Delta f \cdot (t_i+T_u)} \} \]  
\[ = |A(t_i)|^2 \cdot e^{j2\pi\Delta f T_u} \]

where

\[ C_i : \text{correlation value} \]
\[ \Delta f : \text{frequency offset in system} \]
Block Diagram Transceiver

Input Data
- Frequency Interleaving
- QPSK Mapping
- IFFT (Tx)
- FFT (Rx)
- Add Cyclic Extension
- DAC
- RF (Tx)

Output Data
- Frequency Deinterleaving
- QPSK Demapping
- Remove Cyclic Extension
- Timing and Frequency Synchronization
- ADC
- RF (Rx)
Fractional Frequency Offset Estimation

• Frequency Estimation :

  – Fractional part :

  \[ s(t_i) = \text{Re}\{A(t_i)e^{j2\pi(f_c + \Delta f)t_i}\} \]
  \[ = \text{Re}\{\tilde{s}(t_i)e^{j2\pi f_i t_i}\} \quad (1) \]

  \[ \tilde{s}(t_i) \approx A(t_i)e^{j2\pi f_i t_i} \quad (2) \]

  where

  \[ \tilde{S}(t_i) : \text{received data} \]

  \[ t_i : i \text{th sub-carrier duration} \]
Fractional Frequency Offset Estimation

\[ \tilde{s}(t_i + T_u) = A(t_i + T_u) \cdot e^{j2\pi f_{\Delta}(t_i + T_u)} \]

\[ = A(t_i) \cdot e^{j2\pi f_{\Delta}(t_i + T_u)} \]

where

\( A(t_i) \): transmitted data

\( \tilde{S} \): received data

\( T_u \): useful symbol time
Outline

• Introduction
  – Block Diagram Transceiver

• System model

• Simulation results
  – Gaussian
  – Rayleigh

• Code composer studio
  – Gaussian
  – Rayleigh

• Conclusion

• Reference
Fractional Frequency Offset Estimation

\[ C = \sum_{i=1}^{N} c_i = \sum_{i=1}^{N} |A(t_i)|^2 \cdot e^{j2\pi\Delta f T_u} \]

\[ = C_I + jC_Q = |C| \cdot e^{j2\pi\Delta f T_u} \quad (5) \]

\[ \Delta \tilde{f} = \frac{1}{2\pi T_u} \cdot \tan^{-1} \frac{C_Q}{C_I} \quad (6) \]

where

\[ C_I : \text{Real part of correlation value} \]
\[ C_Q : \text{Image part of correlation value} \]
\[ \Delta \tilde{f} : \text{estimated frequency offset} \]
Fractional Frequency Offset Estimation:

\[ \tilde{s}(t_i) \rightarrow A \rightarrow X \rightarrow D \rightarrow \text{Frequency Offset} \rightarrow E \rightarrow \text{Fraction part} \]

\[ \tilde{s}^*(t_i + T_u) \rightarrow B \rightarrow \text{complex conjugate} \rightarrow C \rightarrow \text{Frequency Offset} \]

where

\[ S(t_i)^*: i-th \text{ symbol time of conjugated signal} \]
Criterion of the simulation

\[
\text{mean error} = \frac{1}{N} \sum_{n=0}^{N-1} (\Delta f - \Delta f')
\]

where

\(\Delta f\): frequency offset in system

\(\Delta f'\): estimated frequency offset
Criterion of the simulation

\[ \text{mean square error} = \frac{1}{N} \sum_{n=0}^{N-1} (\Delta f - \tilde{\Delta f})^2 \]

where

\[ \Delta f : \text{frequency offset in system} \]

\[ \tilde{\Delta f} : \text{estimated frequency offset} \]
Gaussian Channel
Channel Impulse Response of Exponential Decaying Rayleigh Fading Channel

\[ h_k = N\left(0, \frac{1}{2} \sigma_k^2\right) + jN\left(0, \frac{1}{2} \sigma_k^2\right) \]
Simulation Results

![Graph showing frequency estimation vs SNR in dB]
Simulation Results

![Graph showing frequency estimation and mean square error vs SNR in dB.]
Simulation Results

![Graph showing BER vs SNR with frequency estimation. The graph has two lines: one for differential and one for guard. The BER decreases as SNR increases.]
Simulation Results (CCS)

Gaussian
Simulation Results (CCS)

Rayleigh
Conclusion

• Simulation analysis
  – mean square error
    • Variance
  – Bit Error Rate
    • Accuracy data in receiver

• Algorithm analysis
  – Fractional Frequency Offset Estimation vs Maximum likelihood (ML) theory
    • Advantage
      – Easy to implementation
    • Disadvantage
      – Fractional frequency offset estimation cannot achieve accuracy of the frequency estimation than ML theory
Reference