The estimation of exponential decay frequency

In the range of the system resonance, the vibration signal takes advantage of the absence of low-frequency mechanical noise. The burst change in the envelope signal for a defect bearing would be the defect impact and possess a high exponential decay frequency. On the contrary, the exponential decay frequency of the envelope signal for a normal bearing would be low. Accordingly, it would be possible to diagnose the bearing defect based on the exponential decay frequency of the envelope signal. A study for the exponential decay frequency of the envelope signal is as follows.

Suppose that the impulse responses due to impacts $d_m(t)$ are completely died out in a time interval between two consecutive contacts and the frequency of $q_m(t) a_{lm}(t)$ is much lower than that of $u_m(t)$, the amplitude of $q_m(t) a_{lm}(t)$ in the time interval could be approximated as a constant $C_{lm}$. Thus, the envelope of an impulse response in the transient-time interval, $\Delta t$, could be expressed as

$$e_{lm}(t) = C_{lm} e^{-\sigma_l t}, \text{ with } 0 \leq t \leq \Delta t. \quad (15)$$

If the number of data points is $k$ over the time interval of the envelope signal in Eq. (14), these points could be expressed in the matrix form,

$$\begin{bmatrix}
\ln(e_{lm}(1)) \\
\ln(e_{lm}(2)) \\
\vdots \\
\ln(e_{lm}(k)) \\
\end{bmatrix}_{k \times 1} = \begin{bmatrix}
1 / 2 f_i & 1 \\
1 / f_i & 1 \\
\vdots & \vdots \\
k / 2 f_i & 1 \\
\end{bmatrix}_{k \times 2} \begin{bmatrix}
\sigma_l \\
\ln(C_{lm}) \\
\end{bmatrix}_{2 \times 1}$$

A simplified expression could be written as

$$[E]_{k \times 1} = [M]_{k \times 2}^T \begin{bmatrix}
\sigma_l \\
\ln(C_{lm}) \\
\end{bmatrix}_{2 \times 1} \quad (16)$$

The solution for $\sigma_l$ and $\ln(C_{lm})$ could be obtained in a linear least squares sense by a simple equation

$$\begin{bmatrix}
\sigma_l \\
\ln(C_{lm}) \\
\end{bmatrix}_{2 \times 1} = \left([M]_{k \times 2}^T [M]_{k \times 2}^{-1} \right) [E]_{k \times 1}, \quad \text{for } k \geq 2 \quad (17)$$

where $T$ denotes the transport of a matrix and $k$ should be properly designated for the adequate estimation. Accordingly, the exponential decay frequency for the $l$th-resonance-mode envelope of an impulse response could be derived. For practical computation of the exponential decay frequency,
frequencies for the envelope of impulse response, the highest peaks of envelope signal derived from Eq. (14) would be picked up for the estimation of \( \sigma \), and an average of 20 estimations for the peaks would be the exponential decay frequency.