

$$y = \frac{1}{1-3x}, \text{ 求 } y^{(n)}$$

解:

$$y = (1-3x)^{-1}$$

$$y' = (-1)(1-3x)^{-2}(1-3x)'$$
$$= (-1)(1-3x)^{-2}(-3)$$

$$y'' = (-1)(-2)(1-3x)^{-3}(-3)(-3)$$
$$= (-1)(-2)(1-3x)^{-3}(-3)^2$$

$$y^{(3)} = (-1)(-2)(-3)(1-3x)^{-4}(-3)^3$$

⋮

$$y^{(n)} = (-1)(-2)(-3)\cdots(-n)(1-3x)^{-(n+1)}(-3)^n$$

$$= (-1)^n (1 \cdot 2 \cdot 3 \cdots n) (1-3x)^{-(n+1)} (-1)^n 3^n$$

$$= (-1)^{2n} n! (1-3x)^{-(n+1)} 3^n$$

$$= \frac{n! 3^n}{(1-3x)^{n+1}}$$