

2.5.2

$$6x^2y'' + 4xy' - y = 0$$

特徵方程式: $6r^2 + (4 - 6)r - 1 = 0$

$$6r^2 - 2r - 1 = 0$$

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 6 \times (-1)}}{2 \times 6} = \frac{1 \pm \sqrt{7}}{6}$$

$$\therefore G.S. \quad y = c_1 x^{\frac{1+\sqrt{7}}{6}} + c_2 x^{\frac{1-\sqrt{7}}{6}}$$

2.5.5

$$x^2y'' + 5xy' + 4y = 0$$

特徵方程式: $r^2 + (5 - 1)r + 4 = 0$

$$r^2 + 4r + 4 = 0$$

$$(r + 2)^2 = 0$$

$$r = -2(\text{重根})$$

$$\therefore G.S. \quad y = (c_1 + c_2 \ln x)x^{-2}$$

2.5.9

$$3x^2y'' + 5xy' + y = 0$$

特徵方程式: $3r^2 + (5 - 3)r + 1 = 0$

$$3r^2 + 2r + 1 = 0$$

$$r = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times 1}}{2 \times 3} = -\frac{1}{3} \pm \frac{\sqrt{2}}{3}i$$

$$\therefore G.S. \quad y = x^{\frac{-1}{3}} \left[c_1 \cos\left(\frac{\sqrt{2}}{3} \ln x\right) + c_2 \sin\left(\frac{\sqrt{2}}{3} \ln x\right) \right]$$