2.1.(三).(a)

$$y_1(t)=e^{\frac{-t^2}{2}}\ ,y_2(t)=e^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds$$

$$y_1{}'(t)=-te^{\frac{-t^2}{2}}\ ,y_2'(t)=-te^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds+1$$

$$y_1''(t)=-e^{\frac{-t^2}{2}}+t^2e^{\frac{-t^2}{2}}\ ,y_2''(t)=-e^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds+t^2e^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds-t$$

$$\because y_1''+ty_1'+y_1=\left(-e^{\frac{-t^2}{2}}+t^2e^{\frac{-t^2}{2}}\right)+t\left(-te^{\frac{-t^2}{2}}\right)+e^{\frac{-t^2}{2}}=0$$

$$y_2''+ty_2'+y_2$$
$$=\left(-e^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds+t^2e^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds-t\right)+t\left(-te^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds+1\right)+\left(e^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds\right)$$
$$=0$$

$\therefore y_1(t)\ ,y_2(t)$是$y''+ty'+y=0$ 之解

2.1.(三).3.(b)

$$W(y_1,y_2)(t)=\begin{vmatrix}y_1(t) & y_2(t)\\ y_1{}'(t) & y_2{}'(t)\end{vmatrix}=y_1(t)y_2{}'(t)-y_1{}'(t)y_2(t)$$

$$=e^{\frac{-t^2}{2}}\left(-te^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds+1\right)-\left(-te^{\frac{-t^2}{2}}\right)\left(e^{\frac{-t^2}{2}}\int_0^t e^{\frac{s^2}{2}}ds\right)$$

$$=-te^{-t^2}\int_0^t e^{\frac{s^2}{2}}ds+e^{\frac{-t^2}{2}}+te^{-t^2}\int_0^t e^{\frac{s^2}{2}}ds=e^{\frac{-t^2}{2}}$$

$$\therefore W(y_1,y_2)(t)=e^{\frac{-t^2}{2}}$$

2.1.(三).3.(c)

$$\begin{cases} y = c_1 y_1 + c_2 y_2 \\ y' = c_1 y_1' + c_2 y_2' \end{cases}$$

$$\begin{cases} y = c_1 e^{\frac{-t^2}{2}} + c_2 e^{\frac{-t^2}{2}} \int_0^t e^{\frac{s^2}{2}} ds \\ y' = -c_1 t e^{\frac{-t^2}{2}} + c_2 \left(-t e^{\frac{-t^2}{2}} \int_0^t e^{\frac{s^2}{2}} ds + 1 \right) \end{cases}$$

$\left[\!\left[\text{代入} \begin{cases} y(0) = 0 \\ y'(0) = 1 \end{cases} \right]\!\right]$

$$\begin{cases} 0 = c_1 \\ 1 = c_2 \end{cases}$$

$$\therefore P.S. \quad y = e^{\frac{-t^2}{2}} \int_0^t e^{\frac{s^2}{2}} ds$$