

2.3.(一).2

$$y'' - 4y' - 5y = 0, y(0) = 2, y'(0) = 1$$

特徵方程式:  $r^2 - 4r - 5 = 0$

$$(r - 5)(r + 1) = 0$$

$$r = 5 \text{ or } r = -1$$

$$\therefore G.S. \quad y = c_1 e^{5x} + c_2 e^{-x}$$

$$\begin{cases} y = c_1 e^{5x} + c_2 e^{-x} \\ y' = 5c_1 e^{5x} - c_2 e^{-x} \end{cases}$$

$$\left[ \text{代入} \begin{cases} y(0) = 2 \\ y'(0) = 1 \end{cases} \right]$$

$$\begin{cases} 2 = c_1 + c_2 \\ 1 = 5c_1 - c_2 \end{cases}$$

$$\begin{cases} c_1 = \frac{1}{2} \\ c_2 = \frac{3}{2} \end{cases}$$

$$\therefore P.S. \quad y = \frac{1}{2} e^{5x} + \frac{3}{2} e^{-x}$$

2.3.(一).7

$$y'' - 10y' + 25y = 0, y(0) = 1, y'(0) = 2$$

特徵方程式:  $r^2 - 10r + 25 = 0$

$$(r - 5)^2 = 0$$

$$r = 5(\text{重根})$$

$$\therefore G.S. \quad y = (c_1 + c_2x)e^{5x}$$

$$\begin{cases} y = (c_1 + c_2x)e^{5x} \\ y' = c_2e^{5x} + 5(c_1 + c_2x)e^{5x} \end{cases}$$

$$\left[ \text{代入} \begin{cases} y(0) = 1 \\ y'(0) = 2 \end{cases} \right]$$

$$\begin{cases} 1 = c_1 \\ 2 = c_2 + 5c_1 \end{cases}$$

$$\begin{cases} c_1 = 1 \\ c_2 = -3 \end{cases}$$

$$\therefore P.S. \quad y = (1 - 3x)e^{5x}$$

2.3.(一).14

$$y'' - y' + y = 0, y(0) = 2, y'(0) = 1$$

特徵方程式:  $r^2 - r + 1 = 0$

$$r = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$r = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$\therefore G.S. \quad y = e^{\frac{1}{2}x} \left[ c_1 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}x\right) \right]$$

$$\begin{cases} y = e^{\frac{1}{2}x} \left[ c_1 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}x\right) \right] \\ y' = \frac{1}{2} e^{\frac{1}{2}x} \left[ c_1 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}x\right) \right] + e^{\frac{1}{2}x} \left[ -\frac{\sqrt{3}}{2} c_1 \sin\left(\frac{\sqrt{3}}{2}x\right) + \frac{\sqrt{3}}{2} c_2 \cos\left(\frac{\sqrt{3}}{2}x\right) \right] \end{cases}$$

$$\left[ \text{代入} \begin{cases} y(0) = 2 \\ y'(0) = 1 \end{cases} \right]$$

$$\begin{cases} 2 = c_1 \\ 1 = \frac{1}{2} c_1 + \frac{\sqrt{3}}{2} c_2 \end{cases}$$

$$\begin{cases} c_1 = 2 \\ c_2 = 0 \end{cases}$$

$$\therefore P.S. \quad y = 2e^{\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

2.3.(三).1

$$y'' + 4y = 0, y(0) = 0, y(\pi) = 0$$

特徵方程式:  $r^2 + 4 = 0$

$$r = \pm 2i$$

$$\therefore G.S. \quad y = c_1 \cos(2x) + c_2 \sin(2x)$$

$$\left[ \text{代入} \begin{cases} y(0) = 0 \\ y(\pi) = 0 \end{cases} \right]$$

$$\begin{cases} 0 = c_1 \\ 0 = -c_1 \end{cases}$$

$$c_1 = 0$$

$$\therefore P.S. \quad y = c_2 \sin(2x)$$