

Ch 3.7 ~ 3.8 偶數題參考答案

Ch 3.7

#4. 由 $4y^2 - xy = 2$,

討論兩邊同時對 x 微分 \Rightarrow

$$\frac{d}{dx}(4y^2 - xy) = \frac{d}{dx}(2)$$

$$\Rightarrow 4 \cdot 2 \cdot y \cdot \frac{dy}{dx} - (1 \cdot y + x \cdot \frac{dy}{dx}) = 0$$

$$\Rightarrow 8y \frac{dy}{dx} - y - x \frac{dy}{dx} = 0$$

$$\Rightarrow (8y - x) \frac{dy}{dx} = y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{8y - x}$$

$$\# 6. \quad \text{由 } \frac{2y-x}{y^2-3} = 5 \quad \text{或} \quad 2y-x = 5(y^2-3)$$

$$\Rightarrow \frac{d}{dx}(2y-x) = 5 \frac{d}{dx}(y^2-3)$$

$$\Rightarrow 2 \frac{dy}{dx} - 1 = 5 \cdot (2y \frac{dy}{dx} - 0)$$

$$\Rightarrow (2-15y^2) \frac{dy}{dx} = 1 \quad \Rightarrow \frac{dy}{dx} = \frac{1}{2-15y^2}$$

$$\# 12. \quad \sqrt{xy} = x-2y, \quad (t, 1)$$

$$\text{求 } \left. \frac{dy}{dx} \right|_{x=4, y=1}$$

$$\frac{d}{dx} \sqrt{xy} = \frac{d}{dx}(x-2y)$$

$$\Rightarrow \frac{1}{2\sqrt{xy}} = \frac{d}{dx}(xy) = 1 - 2 \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{2\sqrt{xy}} \left(y + x \frac{dy}{dx} \right) = 1 - 2 \frac{dy}{dx}$$

$$\Rightarrow y + x \frac{dy}{dx} = 2\sqrt{xy} - 4\sqrt{xy} \frac{dy}{dx}$$

$$\Rightarrow (x + 4\sqrt{xy}) \frac{dy}{dx} = 2\sqrt{xy} - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$$

$$\text{At } x=4, y=1 \Rightarrow \left. \frac{dy}{dx} \right|_{x=4, y=1} = \frac{2\sqrt{4} - 1}{4 + 4\sqrt{4}} = \frac{3}{12} = \frac{1}{4}$$

#14

$$\Rightarrow 3x^2 - 2y + 5 = 0$$

$$\Rightarrow \frac{d}{dx}(3x^2 - 2y + 5) = \frac{d}{dx}(0)$$

$$\Rightarrow 6x - 2 \frac{dy}{dx} + 0 = 0 \quad \Rightarrow \frac{dy}{dx} = \frac{-6x}{-2} = 3x$$

所以在 $(1, 4)$ 点, 切线斜率为 $\frac{dy}{dx} = 3$

Ch 3.8

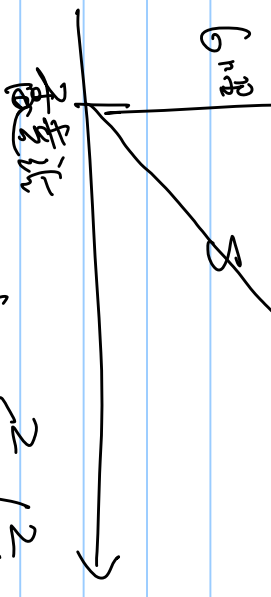
#4

半径为 r 之圆面积为 $A = \pi r^2$, 且 $\frac{dA}{dt} = \frac{d}{dt} \pi r^2$

$$= 2\pi \cdot 2r \frac{dr}{dt} \\ = 4\pi r \frac{dr}{dt}$$

若 $\frac{dA}{dt}$ 之值之固且不为零, 这表示当时间改变, 半径 r 也
随之改变, 因此 $\frac{dA}{dt} = 4\pi r \frac{dr}{dt}$ 不是固之.

#6. $\frac{dS}{dt} = 240 \text{ m}^2/\text{min}$, and $S = 10 \text{ m}^2$



求 $\frac{dx}{dt}$

由 $S^2 = 6^2 + x^2$, 且 $S = 10$ 可求得 $x^2 = 36$ 即 $x = 6$

由 $\frac{d}{dt} S^2 = \frac{d}{dt} (6^2 + x^2)$ 得 $S = 10, x = 6, \frac{dS}{dt} = 240 \text{ m}^2/\text{min}$

$\Rightarrow 2S \frac{dS}{dt} = 2x \frac{dx}{dt}$ (1) $10 \times 240 = 2 \times 6 \frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} = 600 \text{ m}^2/\text{min}$ (水面升速)