2.6.2

$y^{(4)} - 10y'' + 9y = 0$

特徵方程式: $r^4 - 10r^2 + 9 = 0$

$(r-1)(r^3 + r^2 - 9r - 9) = 0$

$(r-1)(r+1)(r^2 - 9) = 0$

$(r-1)(r+1)(r-3)(r+3) = 0$

$r = 1 \;\; or \;\; r = -1 \;\; or \;\; r = 3 \;\; or \;\; r = -3$

$\therefore G.S. \quad y = c_1 e^x + c_2 e^{-x} + c_3 e^{3x} + c_4 e^{-3x}$

2.6.5

$y^{(4)} + 2y'' + y = 0$

特徵方程式: $r^4 + 2r^2 + 1 = 0$

$(r^2 + 1)^2 = 0$

$r^2 = -1$(*重根*)

$r = \pm i$

$\therefore G.S. \quad y = (c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x$

2.6.8

$y''' - 2y'' - y' + 2y = \cos x$

特徵方程式: $r^3 - 2r^2 - r + 2 = 0$

$(r-1)(r^2 - r - 2) = 0$

$(r-1)(r-2)(r+1) = 0$

$r = 1 \;\; or \;\; r = 2 \;\; or \;\; r = -1$

$\therefore y_h = c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x}$

$Let \qquad [\times 2] y_p = A\cos x + B\sin x$

$[\times (-1)] y_p' = -A\sin x + B\cos x$

$[\times (-2)] y_p'' = -A\cos x - B\sin x$

$[\times 1] y_p''' = A\sin x - B\cos x$

$\cos x = (2A - B + 2A - B)\cos x + (2B + A + 2B + A)\sin x$

$$\begin{cases} 4A - 2B = 1 \\ 2A + 4B = 0 \end{cases}$$

$$\begin{cases} A = \dfrac{1}{5} \\ B = -\dfrac{1}{10} \end{cases}$$

$\therefore G.S. \quad y = c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x} + \dfrac{1}{5}\cos x - \dfrac{1}{10}\sin x$