

1. <Problem> Find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$.

$$\vec{F} = -2x\vec{i} + 3xy\vec{j} - 4z\vec{k}$$

<Solution>

$$(a) \nabla \cdot \vec{F} = \frac{\partial(-4x)}{\partial x} + \frac{\partial(3xy)}{\partial y} + \frac{\partial(-4z)}{\partial z}$$

$$= -4 + 3x - 4$$

$$= 3x - 8$$

$$(b) \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2x & 3xy & -4z \end{vmatrix}$$

$$= \vec{i} \left[\frac{\partial(-4z)}{\partial y} - \frac{\partial(3xy)}{\partial z} \right] - \vec{j} \left[\frac{\partial(-4z)}{\partial x} - \frac{\partial(-4x)}{\partial z} \right] + \vec{k} \left[\frac{\partial(3xy)}{\partial x} - \frac{\partial(-4x)}{\partial y} \right]$$

$$= 0\vec{i} + 0\vec{j} + 3y\vec{k}$$