

Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$. [88 清大動機 5(b)]

$$[\text{解}] \text{ 令 } z = e^{i\theta} \Rightarrow \frac{d\theta}{5+3\cos\theta} = \frac{\frac{dz}{iz}}{5+3\frac{z^2+1}{2z}} = \frac{-2idz}{3z^2+10z+3} = \frac{-2idz}{(3z+1)(z+3)}$$

$$\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = \oint_{C_1} \frac{-2idz}{(3z+1)(z+3)}, \text{ 其中 } C_1 \text{ 為圓 } |z|=1$$

$$R_{-\frac{1}{3}} = \lim_{z \rightarrow -\frac{1}{3}} \frac{-2i}{3(z+3)} = -\frac{i}{4}$$

$$\therefore \int_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = 2\pi i \left(-\frac{i}{4}\right) = \frac{\pi}{2}$$