

Evaluate $\oint_C z^6 \sin(1/z) dz$, where C is the circle $|z|=1$ described in the positive direction. [91 中央機械 2(b)]

$$[\text{解}] z^6 \sin(1/z) = z^6 \left(\frac{1}{z} - \frac{1}{3!z^3} + \frac{1}{5!z^5} - \frac{1}{7!z^7} + \dots \right)$$

$$\text{殘數為 } -\frac{1}{7!} = -\frac{1}{5040}$$

$$\therefore \oint_C \frac{(1 - \cos z)^2}{z^7} dz = 2\pi i \left(-\frac{1}{5040} \right) = -\frac{\pi i}{2520}$$