

Solve the differential equation  $y'' + y = -3\sin 2x$ . [106 中正機械甲乙丙 1(b)]

[解] 特徵方程式  $\lambda^2 + \lambda = 0 \Rightarrow \lambda = \pm i$

$$y_h = C_1 \cos x + C_2 \sin x$$

$$\text{Let } y_p = A \sin 2x + B \cos 2x \Rightarrow y_p' = 2A \cos 2x - 2B \sin 2x \Rightarrow y_p'' = -4A \sin 2x - 4B \cos 2x$$

代入原式

$$(-4A \sin 2x - 4B \cos 2x) + (A \sin 2x + B \cos 2x) = -3 \sin 2x$$

$$-3A \sin 2x - 3B \cos 2x = -3 \sin 2x \Rightarrow A = 1, B = 0$$

$$\text{得 } y(x) = y_h + y_p = C_1 \cos x + C_2 \sin x + \sin 2x$$