

Find the solution $y' - 2xy = x^2 + y^2$ [106 中興精密 7(b)]

[解]原式 $\Rightarrow y' = (x + y)^2 \cdots \cdots \cdots$ (i), 令 $u = x + y \Rightarrow u' = 1 + y' \Rightarrow y' = u' - 1$

$$(i) \Rightarrow u' - 1 = u^2 \Rightarrow \frac{du}{dx} = 1 + u^2 \Rightarrow \frac{du}{1 + u^2} = dx \Rightarrow \int \frac{du}{1 + u^2} = \int dx + C$$

$$\tan^{-1} u = x + C \Rightarrow u = \tan(x + C) \Rightarrow x + y = \tan(x + C)$$