

已知方程式 $y'' + 4y' + 4y = 0$ ，試求當 $y(0) = 3, y'(0) = -1$ 之特解。[104 中山環工 2]

[解] 特徵方程式為 $\lambda^2 + 4\lambda + 4 = 0 \Rightarrow (\lambda + 2)^2 = 0 \Rightarrow \lambda = -2, -2$

$$y = k_1 e^{-2x} + k_2 x e^{-2x} \Rightarrow y' = -2k_1 e^{-2x} + k_2 (e^{-2x} - 2x e^{-2x})$$

$$y(0) = 3 \Rightarrow k_1 = 3, y'(0) = -1 \Rightarrow -2k_1 + k_2 = -1 \Rightarrow k_2 = 5$$

$$\text{得 } y = 3e^{-2x} + 5xe^{-2x}$$