

$$(e^{2y} - y \cos xy) dx + (2xe^{2y} - x \cos xy + 2y) dy = 0. \text{ [104 中原機械甲 2]}$$

$$\text{[解]} M = e^{2y} - y \cos xy \Rightarrow \frac{\partial M}{\partial y} = 2e^{2y} - \cos xy - y(-x \sin xy) = 2e^{2y} - \cos xy + xyx \sin xy$$

$$N = 2xe^{2y} - x \cos xy + 2y \Rightarrow \frac{\partial N}{\partial x} = 2e^{2y} - \cos xy - x(-y \sin xy) = 2e^{2y} - \cos xy + xyx \sin xy$$

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} \Rightarrow \text{原式為正合微分方程式}$$

$$u = \int_x M dx + k(y) = \int_x (e^{2y} - y \cos xy) dx + k(y) = xe^{2y} - \sin xy + k(y)$$

$$\frac{\partial u}{\partial y} = N \Rightarrow 2xe^{2y} - x \cos xy + k'(y) = 2xe^{2y} - x \cos xy + 2y \Rightarrow k'(y) = 2y \Rightarrow k(y) = y^2$$

$$\text{解為 } xe^{2y} - \sin xy + y^2 = C$$