

試求解 $(2x+y-7)dx+(x+y-3)dy=0$ 之通解。[104 中山環工 1]

[解] $\begin{cases} 2x+y-7=0 \\ x+y-5=0 \end{cases} \Rightarrow x=2, y=3$, 令 $X=x-2, Y=y-3$, 原式變為

$$(2X+Y)dX+(X+Y)dY=0 \cdots \cdots (i)$$

$$\text{令 } Y=uX \Rightarrow dY=Xdu+udX$$

$$(i) \Rightarrow (2X+uX)dX+(X+uX)(Xdu+udX)=0$$

$$(2+u)dX+(1+u)(Xdu+udX)=0$$

$$[(2+u)+(1+u)u]dX+(1+u)Xdu=0$$

$$(u^2+2u+2)dX+X(1+u)du=0 \Rightarrow \frac{dX}{X} + \frac{1+u}{u^2+2u+2} du = 0$$

$$\int \frac{dX}{X} + \int \frac{1+u}{u^2+2u+2} du = k \Rightarrow \int \frac{dX}{X} + \frac{1}{2} \int \frac{d(u^2+2u+2)}{u^2+2u+2} = k$$

$$2 \ln X + \ln(u^2+2u+2) = 2k \Rightarrow \ln[X^2(u^2+2u+2)] = 2k$$

$$X^2(u^2+2u+2) = C \Rightarrow (uX)^2 + 2X \cdot uX + 2X^2 = C$$

$$Y^2 + 2XY + 2X^2 = C \Rightarrow (y-3)^2 + 2(x-2)(y-3) + 2(x-2)^2 = C$$