

Find the general solution of $(x^2 + y^2)dx = 2xydy$. [106 暨南應光 3]

[解] 令 $y = ux \Rightarrow dy = xdu + udx$, 原式為

$$(x^2 + u^2x^2)dx = 2x \cdot ux(xdu + udx) \Rightarrow (1 + u^2)dx = 2u(xdu + udx)$$

$$[(1 + u^2) - 2u^2]dx = 2uxdu \Rightarrow [-u^2 + 1]dx = 2uxdu$$

$$\frac{dx}{x} = \frac{2udu}{-u^2 + 1} \Rightarrow \int \frac{dx}{x} = \int \frac{2udu}{-u^2 + 1} + k \Rightarrow \ln x = -\ln(1 - u^2) + k$$

$$\ln[x(1 - u^2)] = k \Rightarrow x - xu^2 = C \Rightarrow x - \frac{(xu)^2}{x} = C \Rightarrow x - \frac{y^2}{x} = C$$