

Please find $4y'' - 4y' - 3y = \cos 2x$. [106 中興精密 8]

[解]特徵方程式 $4\lambda^2 - 4\lambda - 3 = 0 \Rightarrow (2\lambda + 1)(2\lambda - 3) = 0 \Rightarrow \lambda = 0, 1, 1 - \frac{1}{2}, \frac{3}{2}$

$$y_h = C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{3}{2}x}$$

$$\text{Let } y_p = A \sin 2x + B \cos 2x \Rightarrow y_p' = 2A \cos 2x - 2B \sin 2x \Rightarrow y_p'' = -4A \sin 2x - 4B \cos 2x$$

代入原式

$$4(-4A \sin 2x - 4B \cos 2x) - 4(2A \cos 2x - 2B \sin 2x) - 3(A \sin 2x + B \cos 2x) = \cos 2x$$

$$(-19A + 8B) \sin 2x + (-8A - 19B) \cos 2x = \cos 2x$$

$$\begin{cases} -19A + 8B = 0 \\ -8A - 19B = 1 \end{cases} \Rightarrow A = -\frac{8}{425}, B = -\frac{19}{425}$$

$$\text{得 } y(x) = y_h + y_p = C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{3}{2}x} - \frac{8}{425} \sin 2x - \frac{19}{425} \cos 2x$$