

Find the general solution for y : $\frac{dy}{dx} = \frac{e^{-x} - y}{x}$. [104 北科大化工 1(b)]

[解]原式 $\Rightarrow xy' = e^{-x} - y \Rightarrow y' + \frac{1}{x}y = \frac{e^{-x}}{x}$

$$F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$y = \frac{1}{F} \left(\int F \cdot \frac{e^{-x}}{x} dx + C \right) = \frac{1}{x} \left(\int e^{-x} dx + C \right) = \frac{1}{x} (-e^{-x} + C)$$