

Solve the ODE $y'' + 4y' + 4y = 6xe^{-2x}$. [105 暨南電機 5]

[解]特徵方程式 $\lambda^2 + 4\lambda + 4 = 0 \Rightarrow (\lambda + 2)^2 = 0 \Rightarrow \lambda = -2, -2$

$$y_h = (C_1 + C_2x)e^{-2x}$$

$$\text{Let } y_p = (Ax^3 + Bx^2)e^{-2x} \Rightarrow y_p' = [-2Ax^3 + (3A - 2B)x^2 + 2Bx]e^{-2x}$$

$$y_p'' = [4Ax^3 + (-12A + 4B)x^2 + (6A - 8B)x + 2B]e^{-2x}$$

代入原式

$$[4Ax^3 + (-12A + 4B)x^2 + (6A - 8B)x + 2B]e^{-2x} + 4[-2Ax^3 + (3A - 2B)x^2 + 2Bx]e^{-2x} + 4(Ax^3 + Bx^2)e^{-2x} = 6xe^{-2x}$$

$$(6Ax + 2B)e^{-2x} = 6xe^{-2x} \Rightarrow A = 1, B = 0$$

$$\text{得 } y(x) = y_h + y_p = (C_1 + C_2x)e^{-2x} + x^3e^{-2x}$$