

解微分方程式 $xy' - 2y = x^3 e^x$ 。 [90 北科大自動化(A)(2)]

[解]原式 $\Rightarrow y' - \frac{2}{x}y = x^2 e^x$

$$F = e^{\int -\frac{2}{x} dx} = e^{-2 \ln x} = x^{-2}$$

$$y = \frac{1}{F} \left(\int F \cdot \frac{e^{-x}}{x} dx + C \right) = \frac{1}{x^{-2}} \left(\int x^{-2} \cdot x^2 e^x dx + C \right) = x^2 (e^x + C)$$