

Find the general solution for the differential equation $(4x^3 + y^3 + 2)dx + xy^2 dy = 0$. [104 交大土木丁
3(a)]

$$[\text{解}] M = 4x^3 + y^3 + 2, N = xy^2 \Rightarrow \frac{\partial M}{\partial y} = 3y^2, \frac{\partial N}{\partial x} = y^2$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{3y^2 - y^2}{xy^2} = \frac{2}{x}$$

$$\mu = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = e^{\ln x^2} = x^2$$

$$u = \int_x \mu M dx + f(y) = \int_x x^2 (4x^3 + y^3 + 2) dx + f(y)$$

$$= \frac{2}{3} x^6 + \frac{1}{3} x^3 y^3 + \frac{2}{3} x^3 + f(y)$$

$$\frac{\partial u}{\partial y} = \mu N \Rightarrow x^3 y^2 + f'(y) = x^2 \cdot xy^2 \Rightarrow f'(y) = 0 \Rightarrow f(y) = 0$$

$$\text{解為 } \frac{2}{3} x^6 + \frac{1}{3} x^3 y^3 + \frac{2}{3} x^3 = C$$