There is a periodic square wave with analytically represented as $f(x)$ function

$f(x) = \begin{cases} -k, & \text{if } -\pi < x < 0 \\ k, & \text{if } 0 < x < \pi \end{cases}$, and $f(x+2\pi) = f(x)$. Please find the Fourier coefficients a_n, b_n and their

series function to present the $f(x)$ function. [106 元智機械 7]

Solution : $f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$,

$a_0 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)dx = 0$,

$a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos nx\,dx = 0$,

$b_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \dfrac{2}{\pi}\displaystyle\int_{0}^{\pi} k \sin nx\,dx = \dfrac{2k}{n\pi}[1-(-1)^n]$

$\quad = \begin{cases} 0, & \text{for even } n \\ \dfrac{4k}{n\pi}, & \text{for odd } n \end{cases} = \dfrac{4k}{(2n-1)\pi}$

$f(x) = \dfrac{4k}{\pi}\displaystyle\sum_{n=1}^{\infty}\dfrac{1}{2n-1}\sin(2n-1)x.$

The graph of the Fourier series is shown as follows: