

Solve the ODE by variable substitution  $x^2y'' - 5xy' + 8y = 2\ln x$ . [106 成大機械 1]

[解] 令  $x = e^t \Rightarrow t = \ln x$  原式為

$$\frac{d}{dt} \left( \frac{d}{dt} - 1 \right) y - 5 \frac{dy}{dt} + 8y = 2t \Rightarrow \frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 8y = 2t \cdots \cdots (i)$$

$$\text{特徵方程式 } \lambda^2 - 6\lambda + 8 = 0 \Rightarrow \lambda = 2, 4 \Rightarrow y_h = C_1 e^{2t} + C_2 e^{4t}$$

$$\text{令 } y_p = At + B \Rightarrow y_p' = A \Rightarrow y_p'' = 0$$

$$(i) \Rightarrow 0 - 6A + 8(At + B) = 2t \Rightarrow A = \frac{1}{4}, B = \frac{3}{16}$$

$$y = y_h + y_p = C_1 e^{2t} + C_2 e^{4t} + \frac{1}{4}t + \frac{3}{16}$$