

Solve the ordinary differential equation  $y' - xy^2 - (1 - 2x)y - x = -1$ . [101 台科大自控 1(1)]

[解]原式  $\Rightarrow y' - x(y^2 - 2y + 1) = y - 1 \Rightarrow y' - x(y - 1)^2 = y - 1 \Rightarrow y' - (y - 1) = x(y - 1)^2$   
 $(y - 1)^{-2}y' - (y - 1)^{-1} = x \dots \dots \dots (i)$  令  $u = (y - 1)^{-1} \Rightarrow u' = -(y - 1)^{-2}y'$ , 代入(i)式  
 $-u' - u = x \Rightarrow u' + u = -x$

$$F = e^{\int dx} = e^x$$

$$u = \frac{1}{F} [\int F \cdot (-x) dx + C] = \frac{1}{e^x} [\int e^x \cdot (-x) dx + C] = e^{-x} [-(xe^x - \int e^x dx) + C]$$
$$= e^{-x} [-(xe^x - e^x) + C] = e^{-x} [(-x + 1)e^x + C] = -x + 1 + Ce^{-x}$$

$$(y - 1)^{-1} = -x + 1 + Ce^{-x} \Rightarrow \frac{1}{y - 1} = -x + 1 + Ce^{-x}$$