

試求 $\frac{dy}{dx} + \frac{2}{x}y + x^4y^3 = 0$ 的通解。[106 南大綠能 6]

$$[\text{解}] \text{原式} \Rightarrow y' + \frac{2}{x}y = -x^4y^3 \Rightarrow y^{-3}y' + \frac{2}{x}y^{-2} = -x^4 \dots\dots\dots(i)$$

$$\text{令 } u = y^{-2} \Rightarrow u' = -2y^{-3}y'$$

$$(i) \Rightarrow -\frac{1}{2}u' + \frac{2}{x}u = -x^4 \Rightarrow u' - \frac{4}{x}u = 2x^4$$

$$F = e^{\int -\frac{4}{x}dx} = e^{-4\ln x} = x^{-4}$$

$$u = \frac{1}{x^{-4}} \left[\int x^{-4} \cdot 2x^4 dx + C \right] = x^4 [2x + C] = 2x^5 + Cx^4$$

$$y^{-2} = 2x^5 + Cx^4$$