

Find the solution of the equation  $(1+x^2)(dy-dx)=2xydx$  for which  $y=1$  when  $x=0$ . [99 高師大電子 1]

$$\text{[解]原式} \Rightarrow dy - dx = \frac{2xy}{1+x^2} dx \Rightarrow y' - 1 = \frac{2xy}{1+x^2} \Rightarrow y' - \frac{2x}{1+x^2} y = 1$$

$$F = e^{\int -\frac{2x}{1+x^2} dx} = e^{-\ln(1+x^2)} = \frac{1}{1+x^2}$$

$$y = \frac{1}{F} \left[ \int F \cdot 1 dx + C \right] = (1+x^2) \left[ \int \frac{1}{1+x^2} \cdot 1 dx + C \right] = (1+x^2) [\tan^{-1} x + C]$$