

解微分方程式 $y' = 2(y\sin 2x + \cos 2x)/\cos 2x$ 。 [90 北科大自動化(A)(1)]

[解]原式 $\Rightarrow 2(y\sin 2x + \cos 2x)dx - \cos 2x dy = 0$

$$M = 2(y\sin 2x + \cos 2x) \Rightarrow \frac{\partial M}{\partial y} = 2\sin 2x$$

$$N = -\cos 2x \Rightarrow \frac{\partial N}{\partial x} = 2\sin 2x$$

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} \Rightarrow \text{原式為正合微分方程式}$$

$$u = \int_x M dx + k(y) = \int_x 2(y\sin 2x + \cos 2x) dx + k(y) = -y\cos 2x + \sin 2x + k(y)$$

$$\frac{\partial u}{\partial y} = N \Rightarrow -\cos 2x + k'(y) = -\cos 2x \Rightarrow k'(y) = 0 \Rightarrow k(y) = 0$$

$$\text{解為 } -y\cos 2x + \sin 2x = C$$