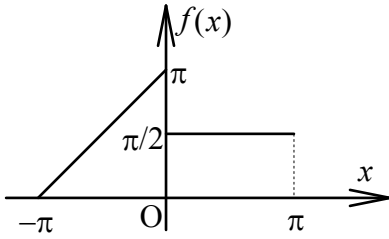


Find the Fourier series of the given function as shown, which is assumed to have the periodic 2π .

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$$[\text{解}] f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \frac{\pi}{2}, & 0 < x < \pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \left[\int_{-\pi}^0 (\pi + x) dx + \int_0^{\pi} \frac{\pi}{2} dx \right] = \frac{1}{\pi} \left[\left(\pi x + \frac{x^2}{2} \right) \Big|_{-\pi}^0 + \frac{\pi}{2} x \Big|_0^{\pi} \right] = \pi$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nxdx = \frac{1}{\pi} \left[\int_{-\pi}^0 (\pi + x) \cos nxdx + \int_0^{\pi} \frac{\pi}{2} \cos nxdx \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n} (\pi \sin nx \Big|_{-\pi}^0 + x \sin nx \Big|_{-\pi}^0 - \int_{-\pi}^0 \sin nxdx + \frac{\pi}{2} \sin nx \Big|_0^{\pi} \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n} (0 + 0 + \frac{\cos nx}{n} \Big|_{-\pi}^0 + 0) \right] = \frac{1 - \cos n\pi}{n^2 \pi} = \frac{1 - (-1)^n}{n^2 \pi}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nxdx = \frac{1}{\pi} \left[\int_{-\pi}^0 (\pi + x) \sin nxdx + \int_0^{\pi} \frac{\pi}{2} \sin nxdx \right]$$

$$= \frac{1}{\pi} \left[-\frac{1}{n} (\pi \cos nx \Big|_{-\pi}^0 + x \cos nx \Big|_{-\pi}^0 - \int_{-\pi}^0 \cos nxdx + \frac{\pi}{2} \cos nx \Big|_0^{\pi} \right]$$

$$= \frac{1}{\pi} \left\{ -\frac{1}{n} [\pi(1 - \cos n\pi) + (0 + \pi \cos n\pi) - 0 + \frac{\pi}{2} (\cos n\pi - 1)] \right\} = -\frac{1}{n\pi} \left[\pi + \frac{\pi}{2} (\cos n\pi - 1) \right]$$

$$= -\frac{1}{n\pi} \left\{ \pi + \frac{\pi}{2} [(-1)^n - 1] \right\}$$

$$f(x) = \frac{\pi}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{1 - (-1)^n}{n^2} \cos nx - \frac{1}{n} \left\{ \pi + \frac{\pi}{2} [(-1)^n - 1] \right\} \sin nx \right\}$$