

Find the Fourier series representation of the square wave which is given $f(x) = \begin{cases} 3, & -\pi \leq x < 0 \\ 5, & 0 < x < \pi \end{cases}$
and $f(x+2\pi) = f(x)$. [86 清大動機 4]

$$[\text{解}] f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nxdx = 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nxdx = \frac{2}{\pi} \int_0^{\pi} k \sin nxdx = \frac{2k}{n\pi} [1 - (-1)^n]$$

$$= \begin{cases} 0, & \text{for even } n \\ \frac{4k}{n\pi}, & \text{for odd } n \end{cases} = \frac{4k}{(2n-1)\pi}$$

$$f(x) = \frac{4k}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x.$$