

Any real matrix \mathbf{A} can be written as the sum of a symmetric \mathbf{R} and a skew-symmetric matrix \mathbf{S} .
Now \mathbf{A} is given below, what are \mathbf{R} and \mathbf{S} ? [86 成大機械 4]

$$\mathbf{A} = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 0 & -1 \\ -3 & 13 & -4 \end{bmatrix}$$

$$[\text{解}] \mathbf{A} = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 0 & -1 \\ -3 & 13 & -4 \end{bmatrix} \Rightarrow \mathbf{A}^T = \begin{bmatrix} 3 & 6 & -3 \\ -4 & 0 & 13 \\ -1 & -1 & -4 \end{bmatrix}$$

$$\mathbf{A} = \frac{\mathbf{A} + \mathbf{A}^T}{2} + \frac{\mathbf{A} - \mathbf{A}^T}{2} = \frac{\mathbf{A} + \mathbf{A}^T}{2} + \frac{\mathbf{A} - \mathbf{A}^T}{2}$$

$$\mathbf{R} = \frac{\mathbf{A} + \mathbf{A}^T}{2} = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 6 \\ -2 & 6 & -4 \end{bmatrix}, \mathbf{S} = \frac{\mathbf{A} - \mathbf{A}^T}{2} = \begin{bmatrix} 0 & -5 & 1 \\ 5 & 0 & -7 \\ -1 & 7 & 0 \end{bmatrix}$$