Prove that Evaluate $\oint_C \dfrac{dz}{(z-a)^n} = \begin{cases} 2\pi i, & n=1 \\ 0, & n=2,3,4,\cdots \end{cases}$, where C is a simple closed curve bounding a region having $z=a$ as interior point. [87 台科大機械 3]

[解]令 C 為圓心 $z=a$ 半徑 1 的圓，則 $z-a=e^{i\theta} \Rightarrow dz=ie^{i\theta}d\theta$

 (1)$n=1$ 時，$\oint_C \dfrac{dz}{z-a} = \oint_C \dfrac{ie^{i\theta}d\theta}{e^{i\theta}} = \int_0^{2\pi} id\theta = 2\pi i$

 (2)$n\neq 1$ 的整數時，$\oint_C \dfrac{dz}{(z-a)^n} = \oint_C \dfrac{ie^{i\theta}d\theta}{e^{in\theta}} = i\oint_C e^{i(1-n)\theta}d\theta = \dfrac{i}{i(1-n)} \cdot e^{i(1-n)\theta}\Big|_0^{2\pi} = 0$