Chapter 16 Non-Stationarity and Unit-Root Test

In stationary time series, shocks will be temporary and over time their effects will be eliminated as the series revert to their long-run mean values. Non-stationary time series will necessarily contain permanent components. A stationary series (correlogram) will die out quickly as the lag-length increases, a non-stationarity time series will not die out for increasing lag length.

Unit roots and spurious regressions

What is a unit root?

Consider the AR(1) model:

\[ y_t = \phi y_{t-1} + e_t \]  

(16.1)

where \( e_t \) is a white-noise process and the stationarity condition is \( |\phi| < 1 \).

\[ y_t - y_{t-1} = y_{t-1} - y_{t-1} + e_t \]
\[ \Delta y_t = e_t \]  

(16.2)

Definition 1  A series \( y_t \) is integrated of order one (denoted by \( y_t \sim I(1) \)) and contains a unit root, if \( y_t \) is non-stationary but is \( \Delta y_t \) stationary.

Definition 2  A series \( y_t \) is integrated of order \( d \) (denoted by \( y_t \sim I(d) \)) if \( y_t \) is non-stationary but is \( \Delta^d y_t \) stationary; \( \Delta y_t = y_t - y_{t-1} \) and

\[ \Delta^2 y_t = \Delta(\Delta y_t) = \Delta y_t - \Delta y_{t-1} \), etc.
Spurious regressions
Most macroeconomic time series are trended and therefore in most cases are non-stationary. The problem with non-stationary or trended data is that the standard OLS regression procedures can easily lead to in correct conclusions. One of the main reasons for taking the logarithm of data before subjecting it to formal econometric analysis.

Suppose we have a series $x_t$ which increases by 10% every period, thus;
\[ x_t = 1.1x_{t-1} \]
if we then take the log of this we get
\[ \log(x_t) = \log(1.1) + \log(x_{t-1}) \]
This series would now be I(1).

More formally, consider the model:
\[ y_t = \beta_1 + \beta_2 x_t + u_t \] (16.3)
A spurious regression usually has a very high $R^2$, t statistic that appear to provide significant estimates, but the results may have no economic meaning whatsoever.

Granger and Newbold (1974) constructed a Monte Carlo analysis generating a large number of $y_t$ and $x_t$ series containing unit roots following the formulas:
\[ y_t = y_{t-1} + e_{yt} \] (16.4)
\[ x_t = x_{t-1} + e_{xt} \] (16.5)
Where $e_{yt}$ and $e_{xt}$ are artificially generated normal random numbers.

When they regressed the various $y_t$s to the $x_t$s as shown in equation (16.3), they surprisingly found that they were unable to reject the null hypothesis of $\beta_2=0$ for approximately 75% of their cases.
They also found that their regressions had very high $R^2$s and very low values of DW statistic.

Smpl @first @first+1
Genr y=0
Genr x=0
Smpl @first+1 @last
Genr y=y(-1)+nrnd
Genr x=x(-1)+nrnd
Scat (r) y x
Smpl @first @last
ls y c x
Granger and Newbold (1974) proposed the following rule of thumb for detecting spurious regressions: IF $R^2 >$ DW-statistic or $R^2 \sim 1$ then the regression must be spurious.

**Explanation of the spurious regression problem**
When the variables become non-stationary then of course we can not guarantee that the errors will be stationary and in fact as a general rule the error itself becomes non-stationary and when this happens we violate the basic assumptions of OLS. If the errors were non-stationary we would expect them to wander around and eventually get large.

**Testing for unit roots**
Testing for the order of integration

**The simple Dickey-Fuller test for units**
Dickey and Fuller (1979, 1981) devised a procedure to formally test for non-stationary. The key insight of their test is that testing for the existence of a unit root.

Simple AR(1) model of the form:

$$y_t = \phi y_{t-1} + u_t$$  \hspace{1cm} (16.9)

We need to examine here is whether $\phi$ is equal to 1. the null hypothesis is $H_0 : \phi = 1$, and the alternative hypothesis is $H_1 : \phi < 1$.

$$y_t - y_{t-1} = \phi y_{t-1} - y_{t-1} + u_t$$

$$\Delta y_{t-1} = (\phi - 1)y_{t-1} + u_t$$  \hspace{1cm} (16.10)

where of course $\gamma = (\phi - 1)$. Then, now the null hypothesis is $H_0 : \gamma = 0$ and the alternative hypothesis is $H_0 : \gamma < 0$, where if $\gamma = 0$ then $y_t$ follows a pure random walk model.

Dickey and Fuller (1979) also propose two alternative regression equation that can be used for testing for presence of a unit root.

The first contain in the random-walk process as in the following equation:

$$\Delta y_{t-1} = \alpha_0 + \gamma y_{t-1} + u_t$$  \hspace{1cm} (16.11)

The second case is to also allow, a non-stochastic time trend in the model, so as to have:

$$\Delta y_{t-1} = \alpha_0 + \alpha_1 t + \gamma y_{t-1} + u_t$$  \hspace{1cm} (16.12)
The augmented Dickey-Fuller (ADF) test for units
As the error term is unlikely to be white noise, Dickey and Fuller extended their test procedure suggesting an augmented version of the test which includes extra lagged terms of the dependent variable in order to eliminate autocorrelation.
The lag length on these extra term is either determined by AIC or BIC.

The three possible forms of the ADF test are given by the following equations:

\[ \Delta y_{t-1} = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t \]  
(16.13)

\[ \Delta y_{t-1} = \alpha_0 + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t \]  
(16.14)

\[ \Delta y_{t-1} = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t \]  
(16.15)

The Phillips-perron test
The distribution theory supporting the Dickey-Fuller tests is based on the assumption that the error terms are statistically independent and have a constant variance.
Phillips-perron (1988) developed a generalization of ADF test procedure that allows for fairly mild assumptions concerning distribution of errors. The test regression for the Phillips-perron (pp) test is the AR(1) process:

\[ \Delta y_{t-1} = \alpha_0 + \gamma y_{t-1} + e_t \]  
(16.16)

When the ADF test corrects for higher order serial correlation by adding lagged differenced terms on the right-hand side, the pp test makes a correction to the t statistic of the coefficient \( \gamma \) from the AR(1) regression to account for the serial correlation in \( e_t \).
The pp statistics are just modifications of the ADF t statistics that take into account the less restrictive nature of the error process.

Unit-root test in EViews
Performing unit-root tests in EViews